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THE SYSTEM OF β LYRAE.[1]

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Of the various theories which have been proposed to explain the light changes of variables, three have been most widely accepted :

First. A body whose surface possesses different degrees of brightness at different places, rotates upon an axis, bringing the differently illuminated portions into view at regular intervals.

Second. A secondary meteoric swarm circles about a primary swarm, so as to pass at regular intervals between the observer and the central swarm. The outlying meteors of the two swarms, colliding with each other at the times of periastron passage, produce also a periodic increase of brightness. A suitable combination of these two causes of light variation will explain a large variety of the light fluctuations observed in variables.

Third. The so-called satellite theory, in which two bodies whose luminous intensities may be either equal or unequal, revolve about each other in orbits whose planes pass near the solar system, and, by the mutual eclipses of the bodies, light changes of such character are produced as to explain some sort of variability.

[1] Read at the conferences held in connection with the dedication of the Yerkes Observatory, Oct. 20, 1897.

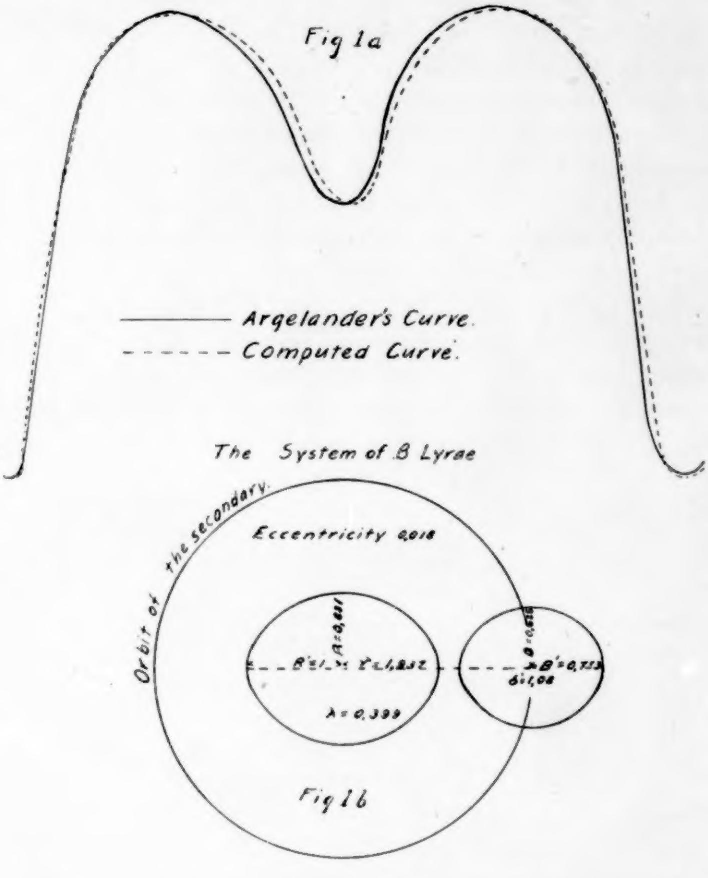
The arbitrariness involved in the fundamental assumptions of theories one and two, appears to me to be an element of weakness of so serious a nature as to compel them to yield to the third theory in all cases where the latter is applicable. By proper assumptions, indeed, regarding the extent, mode of distribution, and periodicity of the spots, or respecting the periodicity and distribution of the meteors within the swarms, as also the relative motions of the swarms themselves, any sort of light change is capable of explanation by the first two theories; but as has been pointed out, their well-nigh universal applicability is in itself an element of weakness, inasmuch as the theories in themselves are in no essential particular an advance on their underlying hypotheses.

It is my purpose to give, in a few words, an account of a recent attempt to represent the light changes of β Lyrae, on the basis of the so-called satellite theory. Argelander's third curve of this star was based upon 1500 careful photometric estimates, extending over a period of nineteen years. An earlier curve by Oudemans and a later by Schoenfeld, show no discrepancies from this curve of a magnitude great enough to affect the discussion appreciably, and, consequently, Argelander's third curve, published in a pamphlet entitled, "De Stella β Lyrae Variabili Commentatio Altera" in 1859, was taken as the basis for the discussion. This well-known curve is represented in the upper portion of the accompanying figure.

INTRODUCTORY.

The variability of this star was discovered by Goodericke in 1784, but aside from the recognition of the fact of its variability and the confirmation of the existence of two unequal minima by this same scientist, nothing further was done in its study until Argelander laid the foundation for a thoroughly scientific and, at the same time, an extremely practical method of studying the light fluctuations of variables. Applying his own method to β Lyrae, Argelander showed the star to have two unequal minima, separated by two practically equal maxima, the entire

cycle of changes being completed in about $12^d\ 22^h\ (=12^d.91)$. At the time of the I Maximum, he found the brightness of the star to rise to the 3.4 magnitude, and to fall after about three



days to a secondary minimum of the 3.9 magnitude, to rise again after three days to the former brightness of the 3.4 magnitude and then, after nearly the same interval, to fall to a primary minimum of the 4.5 magnitude. Or, in other words, the brightness at the maximum corresponds to 12.33 of Argelander's

grades (0.127 mag.); at the primary minimum, to 3.34 grades and at the secondary minimum, to 8.53 grades.

Assuming the light changes to be due to the mutual eclipses of two revolving bodies of unequal brightness, we should have the maxima occurring when the components stand beside each other, without either being eclipsed by the other, and consequently both disks shining full-phase. At the primary minimum, the darker component lies in front of the brighter and cuts off a portion of the latter's light, while at the secondary minimum the bright companion passes before the darker and obscures a part of its inferior brilliance. The relative positions of the components at the chief epochs are shown in the drawing (Fig. 2).

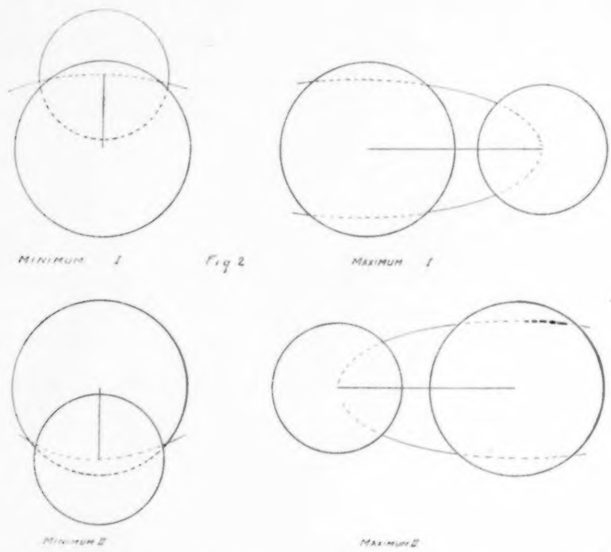


Fig 2

Since the brightness of the "system" is reduced at Minimum I (primary minimum) by 66 per cent. of its greatest brightness and at Minimum II (secondary minimum) by only 36 per cent., it is evident (1), that the disks must be assumed unequally bright or (2), that the orbital eccentricity must be assumed great or finally, that both these circumstances concur.

ECCENTRICITY.

An approximate idea of the magnitude of the eccentricity may be obtained in the following two ways:

First. If we assume the motions of the components to be in conformity to Kepler's laws we have, for the relation of the true and mean anomalies, the well-known equation:

$$v = M + 2e. \sin M + \tfrac{5}{4} e^2 \sin 2M + \quad . \quad . \quad . \quad .$$

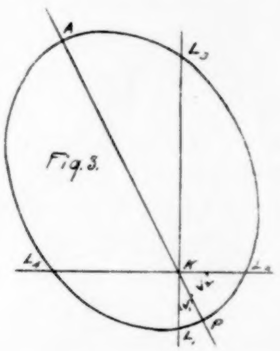
The lengths of the chief intervals of light change, from Argelander's curve are:

Min. I — Max. I	= 3.125 days.
Max. I — Min. II	= 3.250 "
Min. II — Max. II	= 3.167 "
Max. II — Min. I	= 3.368 "

The approximate equality of these intervals points unmistakably to a small orbital eccentricity. Assuming now, the eccentricity to be so small as to render its higher powers negligible, we may shorten the above equation into:

$$(a) \qquad v = M + 2e \sin M.$$

Designating the respective values of v and M at Min. I, Max. I, Min. II and Max. II by v_1, M_1; v_2, M_2; v_3, M_3 and v_4, M_4, and substituting in (a), there result the following four equations:



$$(1) \quad v_1 = M_1 + 2e. \sin M_1 \qquad (3) \quad v_3 = M_3 + 2e. \sin M_3$$

$$(2) \quad v_2 = M_2 + 2e. \sin M_2 \qquad (4) \quad v_4 = M_4 + 2e. \sin M_4$$

Subtracting (1) from (2), (3), and (4) in succession, and noting that $v_2 - v_1 = \frac{\pi}{2}$; $v_3 - v_1 = \pi$; and $v_4 - v_1 = \frac{3\pi}{2}$; we obtain

(5) $$\frac{\pi}{2} = 2\,m_1 + 4\,e.\cos(M_1 + m_1)\sin m_1$$

(6) $$\pi = 2m_2 + 4e.\cos(M_1 + m_2)\sin m_2$$

(7) $$3\frac{\pi}{2} = 2\,m_3 + 4\,e.\cos(M_1 + m_3)\sin m_3,$$

where $$2m_1 = M_2 - M_1 = 3.125\mu = 37^\circ\ 8'.4$$
$$2m_2 = M_3 - M_1 = 6.375\mu = 177^\circ\ 46'.2$$
$$2m_3 = M_4 - M_1 = 9.542\mu = 266^\circ\ 4'.8$$

and $$\mu = 2\,\pi / P, \qquad P = 12.91 \text{ days.}$$

We shall then have but two unknowns in (5), (6) and (7), viz., $e.$ and M_1, and any pair will suffice to determine these magnitudes.

From (5) and (6) we find,

$$(b)\ \begin{cases} M_1 = -\ 30^\circ 28' \\ e = 0.0186, \end{cases}$$

and these values substituted in (1), give

$$v_1 = -31^\circ\ 33'.$$

This satisfies the intervals Max. I—Min. I, and Min. II—Min. I, and requires Min. I to lie $31^\circ.5$ before periastron.

From (5) and (7), there result, similarly:

$$(c)\ \begin{cases} M_1 = 209^\circ 32'4 \\ e = 0.0196 \\ v_1 = 209^\circ 24'. \end{cases}$$

Computing v_1 from Argelander's curve, it will be found that a rigorous satisfaction of the intervals Min. I to Max. I and Min. I to Min. II, requires a backward displacement of Argelander's Max. II by about 4 hours, while the intervals Min. I—Max. I and Min. I—Max. II, used in getting (c), necessitate a backward displacement of Max. I by about 4 hours. Since now, so small a shift in these two chief epochs, corresponds to so large a

change (nearly 180°) in the position of periastron, the eccentricity of the orbit cannot be large. The mean of the two nearly equal values of the eccentricity found above, is 0.0191 ± 0.0033. This value is small enough to justify neglecting its second and higher powers as was done above and thereby vindicates the method of treatment.

Second. It will develop later that the hypothesis of a flattening of one or both bodies must be made. Assuming the bodies to be deformed by reciprocal tidal influence, or by whatever cause, into similar ellipsoids of revolution — a permissible assumption, since such forms are figures of equilibrium — and denoting the ratio of the major and minor axes by "q," so soon as an approximate value of q is known, a superior limit of the ratio of distance of centers, to the larger semi-major axis may be derived. Thus, assuming the bodies to be spheres, with radii equal to the respective semi-minor axes of the ellipsoids, a light curve due to two revolving spheres may be computed. A lower limit for the duration of the eclipse at Min. I, for example, may be read off from this curve. A little reflection will show that the larger q be taken, the shorter the duration of the eclipse will be. Taking then, a value of q greater than that found later to be the approximate value and assuming that the eclipse has not begun until the light curve has fallen considerably, a value for the eclipse-duration will be obtained which is, at all events, small enough, perhaps much too small. q is later found to be 1.2, and assuming it to be 1.3, I find for inferior limit of eclipse-duration, 3 days and 4 hours, which must be at all events, small enough. But the smaller this inferior limit, the larger must be the ratio of distance of centers to the larger semi-major axis. Using the above value $3^d\ 4^h$ for eclipse-duration, a superior limit for this ratio may then be obtained, which will at any rate, be large enough. Taking now, the larger radius as unity and denoting the smaller by κ, the radius vector of the true orbit by r (for this, e may be put $= 0$), one-half the distance between the nearest points of the satellite at the beginning and end of the eclipse, by x, we see from the figure that,

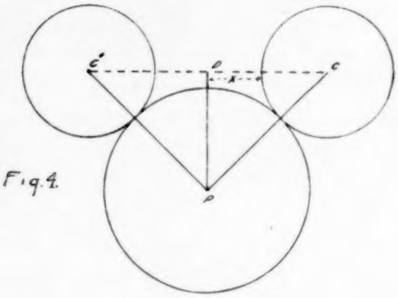


Fig. 4.

$$CPC' \geqq 3.167 \times 27^\circ.887 = 88^\circ\ 20'$$

and hence $\quad CPD \geqq 44^\circ\ 10'.$

Also $\quad r \leqq (x+\kappa)\csc 44^\circ\ 10'$

$$\leqq (x+\kappa)\ 1.438.$$

But since $x \leqq 1$, $\kappa \leqq 1$, we have $r \leqq 2.87$ times the smaller semi-axis of the larger ellipsoid or $= 2.4$ times its larger semi-axis. The distance between centers then, being so small compared with the dimensions of the larger body, the eccentricity could not be large or the masses would necessarily interpenetrate during revolution.

These considerations are sufficient to warrant the assumption of a small orbital eccentricity and to justify the hypothesis, that a first approximation to the orbit may be obtained by making $e = 0$.

FLATTENING.

The necessity of the assumption either that the disks are flattened, or that the bodies are not yet separated, is apparent from the fact that the brightness at the maxima, does not remain constant for any considerable length of time. It has been assumed in what follows that :

(1) The two bodies are distinct and separate.

(2) Both are deformed into ellipsoids of revolution.

(3) The periods of rotation and of revolution are equal to $12^d.91$.

(Possible librations being disregarded.)

Taking the origin of coördinates at the center of the larger ellipsoid and assuming the brightness of the disks uniform, we may readily find from the principles of analytical geometry of space, the equation of the light curve, which may be used while the bodies stand wholly off each other. It was found to be

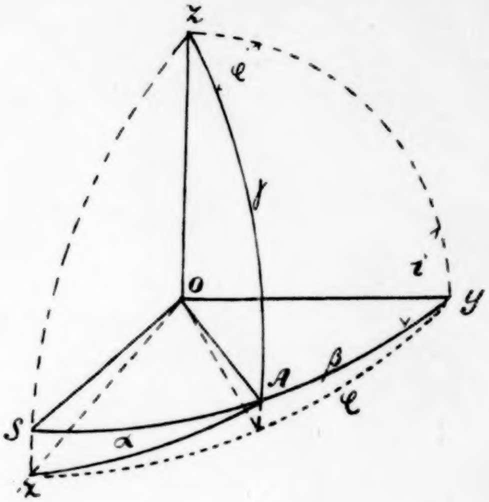


Fig. 5

$$L = \frac{1}{1 + \kappa^2 \lambda} \left\{ \frac{1}{\sqrt{\sin^2 \phi + (R/R')^2 \cos^2 \phi}} + \frac{\kappa_2 \lambda}{\sqrt{\sin_2 \phi + (B/B')^2 \cos^2 \phi}} \right\}$$

Where L denotes the ratio of the combined luminosities of both simultaneous elliptical disks to the combined brightness when the areas of these disks are greatest, *i. e.*, at the instant of the maxima :

λ is the ratio of the intensities of the disks,

κ is the ratio of the corresponding semi-axes of the two ellipsoids.

B', R' and B, R denote respectively the semi-major and semi-minor axes of the smaller and larger ellipsoids, Ox is the sight line and ZOy the tangent plane to the celestial sphere at the system.

ϕ is the longitude of the secondary in its orbit.

As a first approximation q was put equal to $R'/R = B'/B$. From the unsymmetrical character and small magnitude of the final residuals furnished by a comparison of Argelander's curve with the computed curve, it appears later, that nothing can be gained by an attempt to improve this hypothesis by ascribing different degrees of flattening to the two ellipsoids, so long as both bodies be regarded symmetrical and their disks uniformly bright. On this hypothesis the above equation becomes

$$L = \frac{1}{\sqrt{\sin^2 \phi + 1/q^2 \cos^2 \phi}}, \quad \text{where } \phi = \frac{1}{12P}t \quad (t \text{ being in hours}).$$

An approximate value of q was found by computing light curves for various values of q, viz., 1.1; 1.2; 1.25; and 1.3, through a number of symmetrically chosen points before and after the maxima, and deducting the computed values of the ordinates from those of Argelander's curve. The correct value of q should, of course, give a curve whose ordinates, deducted from the corresponding ordinates of Argelander's curve, would be those of a curve due to two revolving spheres, and for a time on either side of the maxima, *i.e.*, while the two components are wholly uncovered, such a curve must run horizontally. The value 1.2 of q gave the following nearly equal ordinates:

10.33; 10.55; 10.76; 10.78; 10.78; 10.79; 10.78; 10.78; 10.76; 10.55; and 10.33.

This value of q was then assumed as a first approximation.

An equation of a light curve applicable during the eclipses was then computed by the process suggested in the drawing, and for Min. I, the necessary equations were found to be:

$$J_l = 1 - \frac{1}{\pi(1+\kappa^2\lambda)}\left\{\phi + \kappa^2(\pi - \phi') - \rho' \sin\phi\right\}$$

$$\rho' = \frac{\rho}{R_l'} = \frac{\sin(\phi \pm \phi')}{\sin\phi'} \text{ according as } \phi'' < 90^\circ \text{ or } \phi'' > 90^\circ,$$

$$\cos\phi = \frac{1 + \rho'^2 - \kappa^2}{2\rho'}$$

$$\sin\phi = \kappa \sin\phi''.$$

Putting $M_I = \pi(1+\kappa^2\lambda)(1-J_I')$.

Then $M_I = \phi + \kappa^2\phi'' - \rho'\sin\phi$ or $M_I = \phi + \kappa^2\phi'' - \kappa\sin(\phi''+\phi)$.

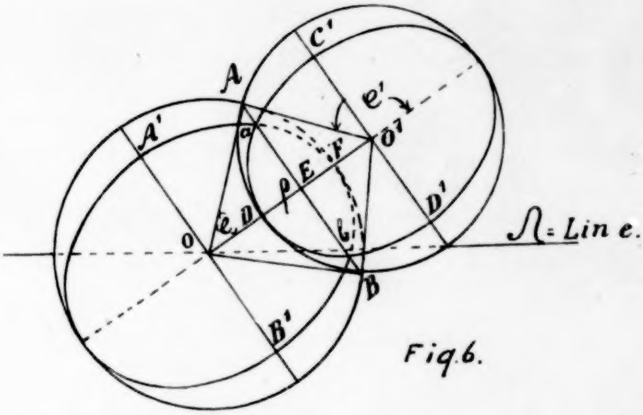


Fig. 6.

The work of computation was somewhat shortened by introducing an additional auxiliary H_I, when $\phi'' > \frac{\pi}{2}$ and by using the foregoing equations only when $\phi'' < \frac{\pi}{2}$. H_I was defined by

$$H_I = \kappa^2\pi - \pi(1+\kappa^2\lambda)(1-J_I'),$$

Whereupon

$$H_I = \kappa^2\phi' - \phi - \rho'\sin\phi, \text{ or } H' = \kappa^2\phi' - \phi + \kappa\sin(\phi' - \phi)$$

and similarly, the equations for Min. II were found to be:

$$J_{II}' = 1 - \frac{\lambda}{\pi(1+\kappa^2\lambda)}\left\{\phi + \kappa^2(\pi - \phi') - \rho'\sin\phi\right\}$$

$$M_{II} = \pi\frac{1+\kappa^2\lambda}{\lambda}\left\{1 - J_{II}'\right\}, \text{ or } M_{II} = \phi + \kappa_2\phi'' - \rho'\sin\phi$$

$$H_{II} = \kappa^2\pi - \pi\frac{1+\lambda\kappa^2}{\lambda}(1-J_{II}'), \text{ or } \kappa^2\phi'' - \phi + \rho'\sin\phi.$$

The remaining formulæ hold without modification for both minima. J_I' and J_{II}' are the ratios of the combined instantaneous brightness to the combined brightness due to the sum of the two instantaneous disks shining full-phase during the eclipses at Min. I and Min. II respectively.

J_I and J_{II} are the ratios of the same brightness to that due to the sum of the full disks, when these disks are of maximum area.

The relation between J_I' and J_I on the one hand, and J_{II}' and J_{II} on the other, were then derived and found to be

$$J_I = fJ_I' \text{ and } J_{II} = fJ_{II}', \text{ where } f = R_i'/R = B_i'/B$$
$$= \frac{1}{\sqrt{\sin^2\beta + q^2\cos^2\beta}},$$

and $\beta = \phi - \frac{\pi}{2}$ denotes the longitude in the orbit, counted from a point 90° ahead of the origin of ϕ. (For undefined symbols see Fig. 6.)

Designating by $\beta + \frac{\pi}{2}$, the true anomaly in the real orbit, and by α that in the apparent orbit, both counted from the node, and calling ρ and r the radii vectores in the apparent and true orbits respectively, we have

$$r.\sin\beta = \rho\cos\alpha \text{ and } \tan\alpha = \cos i\cot\beta,$$

wherefore $$\rho^2 = r^2.\sin^2\beta + r^2.\cos^2 i.\cos\beta.$$

Differentiating the formulæ for Min. I and reducing, we find

$$\delta\phi = \frac{\delta M_I}{2\kappa t g\phi'' \sin(\phi'' + \phi)} \text{ and } \delta\phi = \frac{\delta H_I}{2\kappa t g\phi' \sin(\phi' - \phi)},$$

and when $\kappa > 1$, as will be found to be the case later, the preceding equations become

$$M_I = \phi + \kappa^2\phi' - \kappa\sin(\phi + \phi')$$
$$H_I = \phi - \kappa^2\phi' + \kappa\sin(\phi - \phi')$$
$$\rho' = \frac{\kappa\sin(\phi - \phi')}{\sin\phi}$$
$$\delta\phi = \frac{\delta H_I}{2\kappa t g\phi' \sin(\phi - \phi')}.$$

J was obtained by subtracting the ordinate of Argelander's curve for the selected instants from the mean of the maximum ordinates, calling this difference ΔG, and computing J from the equation $\log J = 0.051\,\Delta G$, which is readily derived from Pogson's scale, together with the value of Argelander's grade. The

values of M_{I}, M_{II}, H_{I}, H_{II}, are computed directly from values of J obtained from the above curve, and then an approximate value of ϕ, interpolated from tables computed from the *M's* and *H's* with ϕ as an argument, are then corrected by the above differential formulæ. Thus it is possible by these formulæ to compute a light curve from known elements, and also to solve the converse problem. To compute these tables the values of κ were required. κ being unknown, an approximation of its value was obtained thus: calling b_m the brightness of a star of magnitude m, and b_M that of a star of magnitude M, by Pogson's scale

$$\log\left(\frac{b_m}{b_M}\right) = \log J = 0.4 \triangle M,$$

where $\triangle M$ is the difference of the brightness in stellar magnitudes. From this we have

$$\frac{\text{Brightness at Min. II}}{\text{Brightness at Min. I}} = e = 1.8536$$

$$\frac{\text{Brightness at Max.}}{\text{Brightness at Min. I}} = m = 2.0123.$$

The first of these gives

$$(d) \qquad \frac{1 + \kappa^2 \lambda - a \kappa^2 \lambda}{1 - a \kappa^2 + \kappa^2 \lambda} = e$$

and the second

$$(e) \qquad q\, \frac{1 + a \kappa^2 + \kappa^2 \lambda}{1 - a \kappa^2 + \kappa^2 \lambda} = m,$$

where a denotes the portion of the disks common to both bodies at the middle of the eclipses.

By a few simple transformations of these equations, it may be readily seen that

$q \geqq 1.0205$, *i. e.* the disks must be flattened, and $0.8323 \geqq \kappa \geqq 1.5241$.

The values of κ selected, were then

0.8323, 0.9049, 1.0000, 1.2883 and 1.5241

and a table such as was mentioned above was computed with the argument ϕ or ϕ' according as $\kappa < 1$, or $\kappa > 1$.

The values of r, for various points before and after the minima, with a correct hypothesis for κ and i, the inclination, must all be approximately equal, since they are computed on the hypothesis of a circular orbit. $\kappa = 1.5241$ gave a fair approach to an agreement in the various values, while the other values of κ gave widely discordant results for r. For some reasons it was more convenient to have $\kappa < 1$, and hence, the larger radius was assumed unity and the necessary alterations in the formulæ were made to suit this hypothesis. The only possible assumption which could be made for i was shown to be $\frac{\pi}{2}$, since this gave the various values of r more nearly equal than any other assumption for it. λ was found to be 0.3353 from (d) and (e). Recapitulating then, the following values were taken as first approximations:

$$e = 0,\ i = \frac{\pi}{2},\ r = 1.8344,\ \kappa = 0.6561,\ \lambda = 0.3353 \text{ and } q = 1.2.$$

By differentiating the above formulæ and combining the results, the following differential equations for correcting the approximate circular elements were derived:

$$\text{(I)}\quad \pi(\lambda + \kappa^2)\, dJ_I = 2\kappa K_I\, d\kappa + \frac{2\rho \sin\phi'}{r}\, dr + r^2 \frac{\cos^2\beta \sin\phi'}{\rho}\, d(i'^2)$$

$$\left(i' = \frac{\pi}{2} - i\right),$$

where $K_I = A J_I f^2 \cos^2\beta + B(f - J_I) - f\phi - C\rho f^2 \cos^2\beta \sin\phi'$

and $A = \pi q \lambda, \quad B = \pi\left(1 + \frac{\lambda(1-\lambda)}{\kappa^2}\right), \quad C = \frac{2q3}{m}$

q must not be included in this equation since it depends on κ by equation (e). The equation applies only during Min. I. For Min. II,

$$\text{(II)}\quad \frac{\pi(\lambda + \kappa^2)}{\lambda}\, dJ_{II} = 2\kappa K_{II}\, d\kappa + \frac{2\rho \sin\phi'}{r}\, dr + \frac{r^2 \cos^2\beta \sin\phi'}{\rho}\, d(i')^2$$

$$K_{II} = \frac{A J_{II} f^2 \cos^2\beta}{\lambda} + \pi(f - J_{II}) - C\rho f^2 \cos^2\beta \sin\phi'.$$

The coefficients of (I) and (II) were computed for 14 points selected symmetrically along the curve, with the approximate values given above. dJ_I and dJ_{II} were obtained by comparing corresponding ordinates of Argelander's curve with those of a light curve computed from the preceding approximate values. 14 observation equations lead to the following values:

$$d\kappa = +0.3586, \quad dr = +0.0999, \quad \text{and } d(i'^2) = -0.0434$$

i' being here imaginary, but numerically small, it was called 0, and $d\kappa$ being quite large, dq was introduced in its stead, since small changes in the fundamental data do not affect q so greatly as r, and the equations again solved, gave the corrections, $dq = -0.0007$, and $dr = -0.0889$, and the corrected values were then $q = 1.1993$ and $r = 1.8955$, and from (d) and (e), $\kappa = 0.7580$ and $\lambda = 0.4023$. The probable error was, of course, somewhat increased by dropping i', being in the latter case ± 0.1 of a "brightness." These values were regarded as the most probable on the assumption of a circular orbit.

By means of these values and differential equations, which were derived for correcting a circular into an elliptical orbit of small eccentricity, a set of 48 observation equations was obtained, for as many points of the light curve, and from them the following elliptical elements were obtained:

$T = 1855$ Jan $13^d\ 6^h.35$	P $= 12.91$ days
$a = 1.937$	$\mu = 27^\circ.887$
$i = 90^\circ$	$\kappa = 0.7528$
$\eta = 94^\circ\ 53'$ from node	$\lambda = 0.399$
$\Omega = 0^\circ$ (assumed)	$q = 1.203$
$\tau = 6^d\ 15^h.4$ after Min. I.	

Both e and i' were included in the equation and i' was again imaginary but very small.

Using the equations derived for computing the light curve for elliptical motion, the results of computation are comprised in the columns of the table given below. The columns headed J_R and J_B contain the computed and observed ordinates respectively, and those headed ΔJ_{B-R}, the corresponding residuals

from Argelander's curve. The computed curve is shown in (Fig. 1_a) drawn in a dotted line beside Argelander's curve drawn in a full line. The agreement is quite close.

TABLE OF COMPUTED AND OBSERVED BRIGHTNESS.

t	Minimum I			Minimum II		
	J_B	J_R	δJ_{B-R}	J_B	J_R	δJ_{B-R}
−72	0.9963	0.9956	+0.0007	0.9918	0.9970	−0.0052
−66	0.9870	0.9852	+0.0018	0.9836	0.9945	−0.0109
−60	0.9683	0.9701	−0.0018	0.9696	0.9833	−0.0137
−54	0.9524	0.9513	+0.0011	0.9490	0.9672	−0.0182
−48	0.9147	0.9270	−0.0123	0.9258	0.9482	−0.0224
−42	0.8732	0.8849	−0.0117	0.8995	0.9217	−0.0222
−36	0.8209	0.8268	−0.0059	0.8680	0.8886	−0.0206
−30	0.7296	0.7525	−0.0229	0.8306	0.8494	−0.0188
−24	0.5836	0.6019	−0.0183	0.7836	0.8048	−0.0212
−18	0.4336	0.4993	−0.0657	0.7246	0.7558	−0.0312
−12	0.3661	0.4275	−0.0614	0.6674	0.7044	−0.0370
− 6	0.3484	0.3487	−0.0003	0.6433	0.6542	−0.0109
0	0.3433	0.3433	+0	0.6365	0.6368	−0.0003
+ 6	0.3499	0.3488	+0.0011	0.6472	0.6372	+0.0100
+12	0.3988	0.4275	−0.0287	0.6762	0.6689	+0.0073
+18	0.5306	0.5591	−0.0285	0.7340	0.7203	+0.0137
+24	0.6572	0.6624	+0.0052	0.7978	0.7716	+0.0262
+30	0.7644	0.7528	+0.0116	0.8498	0.8289	+0.0209
+36	0.8416	0.8266	+0.0150	0.8931	0.8622	+0.0309
+42	0.8857	0.8845	+0.0012	0.9255	0.8997	+0.0258
+48	0.9234	0.9268	−0.0034	0.9524	0.9307	+0.0217
+54	0.9537	0.9508	+0.0029	0.9627	0.9545	+0.0082
+60	0.9732	0.9694	+0.0038	0.9881	0.9732	+0.0149
+66	0.9870	0.9847	+0.0023	0.9977	0.9877	+0.0100
+72	0.9940	0.9952	−0.0012	1.0049	0.9970	+0.0079

With the help of an eccentricity, therefore, the residuals are somewhat reduced, and, considering the errors necessarily attaching to photometric estimates, the curve of Argelander may be regarded as sufficiently well represented.

The eccentricity results again almost the same as before, so that at the epoch 1855, the eccentricity did not differ materially from 0.02.

Since the periastron lies near Min II, the systematic deviation of the computed from Argelander's curve, the former lying above the latter, before, and below it, after Min II, it is highly probable, that while the secondary rounds periastron, very con-

siderable augmentation of brightness due to deformations of the disks, to internal friction, etc., occurs. Because of inertia, these effects could not immediately show themselves, so that the real curve would lie below the computed mean before Min II and above it after Min II, as is represented in the figure.

Feeling somewhat suspicious that any one of several sets of elements lying near those just given might represent observations equally well, it seemed worth while to test whether this same set of elements would result, if one of the circular elements upon which the elliptical ones were based, were given an arbitrary change and then, by repeated applications of the method of least squares, the elements were again corrected by the observations. An arbitrary change of -0.01 was given to κ and of -0.07 to r and after four adjustments giving ever smaller probable errors, the former values were again obtained.

It may therefore be inferred that the above elements are the most probable. An interesting fact arising during the latter process of adjustment, was that one set of elements giving a probable error of nearly the same magnitude as the final set, gave the distance between the centers $= 1.80$ and the sum of the radii equal to 1.82, *i. e.*, the components are not yet separated. This fact in connection with the low mean density of the system points to the nebulous condition of the star. The indications are then, either that the companions are not yet separate, but in the act of separation, or that if separate, their separation has taken place comparatively recently. In either case we seem to have here the first concrete example of a world in the act of being born.

SPECTROSCOPIC CONSIDERATIONS.

Treating by the method of Rambaut, published in *Mon. Not.* **51**, 316, the observations of Bélopolsky made between Sept. 23 and Nov. 26, 1892, and published in *Melanges math. et astronomiques*, *t.* VII, *l.* 3, I find,

$$V = -0.8 \text{ kilometers per second}$$
$$T = 26.93 \text{ September } 1892$$

$$e = 0.108$$
$$\omega = 79^\circ\ 17' \text{ from node}$$
$$a \sin i = 15836000 \text{ kilometers}$$
$$P = 12^d.91 \text{ from light period.}$$

Lockyer observes the relative displacements of three lines and finds velocities as follows:

$$H\gamma = 155.0 \text{ miles}$$
$$H\delta = 154.0 \text{ miles}$$
$$\lambda_{4095} = 158.0 \text{ miles.}$$

The mean of these values is 155.7 miles and from a private letter of Professor Lockyer, I find it belongs to the epoch, August 24.46, 1893. Correcting for the motion of the Earth and reducing to kilometers, there results for the relative velocity in the line of sight 259.8 kilometers.

Bélopolsky's measures give for the diameter of the absolute orbit of one of the components 31672000 kilometers. From the absolute orbit furnished by Bélopolsky's observations, the velocity for the above date was found to be $\frac{1}{3.168}$ of the above relative velocity. The semi-major axis of the relative orbit is then $A = 3.168 / 2 \times 31672000 = 50175000$ kilometers (calling $i = \frac{\pi}{2}$) and for the ratio of the masses

$$a / A = m / (m + M), \text{ or } m / M = \frac{1}{2.168}\ .$$

Assuming that the F-line observed by Bélopolsky was produced by the smaller component, we have

$$A / a = (M + m) / m = 1 + \delta' \kappa^3,$$
$$(\delta' = \text{ratio of densities of components}),$$

and assuming his line to be produced by the larger, there results,

$$A / a = (M + m) / m = 1 + 1 / \delta' \kappa^3.$$

Using the values of a / A and κ given above, we find

$$\delta' = 5.083, \text{ or } = 1.081.$$

This furnishes a means of deciding which of the components produced Bélopolsky's F-line. Since it is quite improbable that one of the two bodies, so near to each other as is the case here, should have a density 5.083 times that of the other, the latter

value is assumed to be the correct one, and hence, that Bélopolsky's F-line was due to the larger component.

Using now the well known relation

$$M + m = A^3 / P^2$$

and substituting the values of A in astronomical units and of P in years, there result,

$$M + m = 30.56 \text{ solar masses}$$

and since $m / M = 1 / 2.168,$

$$M = 20.91 \text{ solar masses}$$

and $m = 9.65$ solar masses.

Calling S the solar mass, H the solar radius, R' and R, the semi-major and semi-minor axes of the minor ellipsoid, δ_1 the density of the larger companion in terms of the solar density and δ_2 of the smaller,

$$\delta_1 = (M / S)\, q^2\, (H / R')^3, \qquad (q = R' / R \text{ as before})$$

and substituting former values,

$$\delta_1 = 0.00058.$$

But

$$\delta_2 = 1.08\, \delta_1 \;\; (\delta' = 1.08)$$

hence

$$\delta_2 = 0.00063.$$

The mean density of the system is then somewhat less than the density of air, *i. e.*, comparable to nebular density. It appears then that β Lyrae furnishes us a concrete illustration of the actual existence in space of a Poincaré figure of equilibrium.

Using the chief epochs of Lindemann's curve constructed from Plassmann's observations, the following equations result:

$$\cos (M_1 + 49^\circ\ 57') = -0.0563 / e = -3.204 \text{ (with Argelander's } e)$$
$$\cos (M_1 + 93^\circ\ 52') = -0.0338 / e = -1.920$$
$$\cos (M_1 + 136^\circ\ 23'.5) = -0.01771 / e = -1.010.$$

The impossibility of these relations, on the hypothesis that e is not greater than it was in 1855, requires us to infer that since Argelander's time the eccentricity of the system has grown larger. Comparing the spectroscopic with the photometric

observations, it is also seen that a large motion of the line of apsides has occurred since 1855, though its precise amount can scarcely be ascertained with any considerable degree of certainty.

The following results, therefore, seem to be quite clearly indicated by the preceding discussion :

1. The photometric estimates of β Lyrae's variability may be explained easily within the limits of the errors of these estimates by the aid of the satellite theory.

2. The bodies may be regarded as similar ellipsoids of revolution.

3. The orbit of the secondary body is nearly circular and its plane passes almost exactly through the Sun.

4. The common flattening of the ellipsoids differs but little from 0.17 and, aside from librations, the periods of rotation and revolution are equal.

5. The larger body is about 0.4 as bright as the smaller.

6. The distance of centers is extremely small, about 1 7/8 of the semi-major axis of the larger ellipsoid.

7. From Lindemann's chief epochs for 1892 the orbital eccentricity of the system must have increased from 1855. Bélopolsky's spectroscopic observations indicate the same.

8. The motion of the center of gravity of the system with respect to the Sun is very small.

9. The semi-major axis of the orbit of the companion is about 50,000,000 kilos.

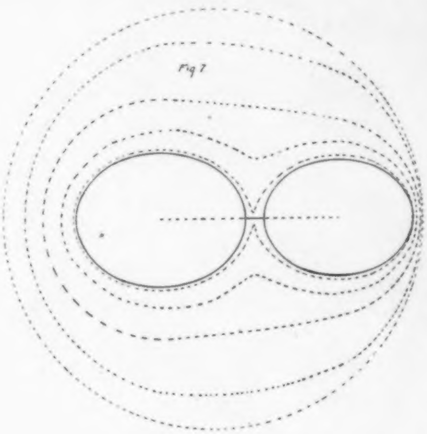
10. The mass of the larger body is 21 times, and of the smaller 9.5 times the solar mass.

11. The densities of the companions are nearly the same.

12. The mean density of the system is comparable with atmospheric density, or the "system" (for such, I think, it must now be called), is in a nebulous condition.

13. In conclusion, it may be said that the spectroscopic and photometric observations, which were available to me for the foregoing discussion, so far from being widely discordant, as some have thought, agree with each other remarkably closely.

The strong absorption lines in the spectrum of this star point to the presence of a powerfully absorbing atmospheric envelope about the nuclei of the masses. From the dynamical theory of gases we know that such an atmospheric layer would arrange itself about the combined mass of the system so that portions of equal density would be in equipotential surfaces. These surfaces would, in the immediate vicinity of the surfaces of the masses, conform somewhat closely to the surface of the bodies, but they would rapidly lose the abrupt curvatures at the surface and become more and more nearly spherical. A rough attempt to represent this is shown in the subjoined figure.



The equipotential surfaces, shown by dotted lines, would dispose themselves symmetrically about the center of gravity of the system as a center, and if the density of the larger companion were very materially greater than that of the smaller, the center of gravity would lie far within the larger companion. It might even happen that the center of gravity of the two bodies should lie beyond the geometrical center of the larger ellipsoid. Where it would lie would depend wholly upon the density of the various parts of the mass of the two bodies. It is then readily seen that the atmosphere might be, and probably is, so arranged as to permit the remote end of the smaller

to shine through a shallow layer of it and thereby to permit the smaller to appear brighter, even though it might be intrinsically darker than the larger body. No violence is done to theory, at all events, by such an assumption. This distribution of the atmosphere would also explain the absorption bands, which are seen in the spectrum of this star more distinctly at Min I than at Min II. The continuous spectrum, which would be produced most distinctly by the smaller body, must at the same time appear fainter. This accords with spectroscopic observations also.

Although some of the ideas given above may seem a little venturesome, let it be remembered that the peculiar character of the observations of this star leads one to expect an explanation of a somewhat unusual nature.

That the ellipsoids are similar is, of course, an arbitrary assumption.

In conclusion, let it be observed that an attempt at a formal representation of the condition of things prevailing in the system of β Lyrae, leads to the assumption of a single body (such as Poincaré's or Darwin's figures of equilibrium). The above has, of course, only a formal significance, but on account of the poverty of observational material at my disposal an attempt to push the discussion farther on a mathematical basis could not have proved profitable. It is believed, however, that the discussion may help us to orient our views with regard to this wonderfully interesting star. Fig. 1*b* represents the most probable relative dimensions of the bodies and orbit of the system, as based on Argelander's photometric estimates up to 1859. Professor Pickering has kindly offered to place all the earlier estimates of β Lyrae's brightness at the writer's disposal, and it is the latter's intention to make a full investigation and discussion of them at the earliest possible date.

THE ALGOL VARIABLE $+17^\circ 4367$. W DELPHINI.

By Edward C. Pickering.

An ephemeris of the times of heliocentric minima of the Algol variable, W Delphini, for the years 1896 and 1897 will be found in the Astrophysical Journal **3**, 200; **4**, 320. The formula employed is J. D. $2412002.500 + 4.8064$ E. A continuation of this ephemeris for the year 1898 is given below in Table I. The first column gives the number of the epoch, the second the Julian Day and fraction after subtracting the constant number 2410000. The observations of the last year indicate that the period of this variable, like those of some, if not all, other variables of the Algol type, is not constant. The deviation from the ephemeris has now become so large, nearly one hour at the beginning of 1898, that corrected times of minima expressed in calendar days, and in hours and minutes of Greenwich Mean Time are given in the third and fourth columns. The amount of the correction is $0^{m}.45$ (E–357) which closely represents the observations for the last two years, and can readily be applied to the preceding ephemerides. While, therefore, the second column should be used in comparing the results of observation with those of preceding years, the fourth column in connection with the light curve (this Journal, **4**, 323) will indicate more closely the times at which observations should be made. Before the next annual ephemeris is published I hope to discuss the observations so far made of this star, and to indicate the nature of the variations in period.

TABLE I.

EPHEMERIS FOR 1898.

E.	J. D.	Date	Corrected G. M. T.	E	J. D.	Date	Corrected G. M. T.
477	4295.1528	Jan. 5	2^h 46^m	515	4477.7960	July 6	17^h 55^m
478	4299.9592	" 9	22 07	516	4482.6024	" 11	13 16
479	4304.7656	" 14	17 28	517	4487.4088	" 16	8 37
480	4309.5720	" 19	12 48	518	4492.2152	" 21	3 57
481	4314.3784	" 24	8 09	519	4497.0216	" 25	23 18
482	4319.1848	" 29	3 30	520	4501.8280	" 30	18 39
483	4323.9912	Feb. 2	22 50	521	4506.6344	Aug. 4	13 59
484	4328.7976	" 7	18 11	522	4511.4408	" 9	9 20
485	4333.6040	" 12	13 32	523	4516.2472	" 14	4 41
486	4338.4104	" 17	8 53	524	4521.0536	" 19	0 02
487	4343.2168	" 22	4 14	525	4525.8600	" 23	19 22
488	4348.0232	" 26	23 34	526	4530.6664	" 28	14 43
489	4352.8296	Mar. 3	18 55	527	4535.4728	Sept. 2	10 04
490	4357.6360	" 8	14 16	528	4540.2792	" 7	5 25
491	4362.4424	" 13	9 36	529	4545.0856	" 12	0 45
492	4367.2488	" 18	4 57	530	4549.8920	" 16	20 06
493	4372.0552	" 23	0 18	531	4554.6984	" 21	15 27
494	4376.8616	" 27	19 39	532	4559.5048	" 26	10 48
495	4381.6680	Apr. 1	15 00	533	4564.3112	Oct. 1	6 09
496	4386.4744	" 6	10 20	534	4569.1176	" 6	1 30
497	4391.2808	" 11	5 41	535	4573.9240	" 10	20 50
498	4396.0872	" 16	1 02	536	4578.7304	" 15	16 11
499	4400.8936	" 20	20 23	537	4583.5368	" 20	11 32
500	4405.7000	" 25	15 44	538	4588.3432	" 25	6 53
501	4410.5064	" 30	11 04	539	4593.1496	" 30	2 14
502	4415.3128	May 5	6 25	540	4597.9560	Nov. 3	21 34
503	4420.1192	" 10	1 46	541	4602.7624	" 8	16 55
504	4424.9256	" 14	21 07	542	4607.5688	" 13	12 16
505	4429.7320	" 19	16 27	543	4612.3752	" 18	7 37
506	4434.5384	" 24	11 48	544	4617.1816	" 23	2 58
507	4439.3448	" 29	7 09	545	4621.9880	" 27	22 18
508	4444.1512	June 3	2 30	546	4626.7944	Dec. 2	17 39
509	4448.9576	" 7	21 51	547	4631.6008	" 7	13 00
510	4453.7640	" 12	17 11	548	4636.4072	" 12	8 20
511	4458.5704	" 17	12 32	549	4641.2136	" 17	3 41
512	4463.3768	" 22	7 53	550	4646.0200	" 21	23 02
513	4468.1832	" 27	3 13	551	4650.8264	" 26	18 23
514	4472.9896	July 1	22 34	552	4655.6328	" 31	13 43

HARVARD COLLEGE OBSERVATORY,
CAMBRIDGE, MASS., November 27, 1897.

OF ATMOSPHERES UPON PLANETS AND SATELLITES.[1]

By G. Johnstone Stoney.

INTRODUCTION.

The present writer began early in the sixties to investigate the phenomena of atmospheres by the kinetic theory of gas, and in 1867 communicated to the Royal Society a memoir,[2] which pointed out the conditions which limit the height to which an atmosphere will extend, and in which it was inferred that the gases of which an atmosphere consists attain elevations depending on the masses of their molecules, the lighter constituents overlapping the others. This was disputed at the time, on account of its supposed conflict with Dalton's law of the equal diffusion of gases;[3] but physical astronomers now recognize its truth.

On December 19, 1870, the author delivered a discourse before the Royal Dublin Society, which was the first of the series of communications, of which an account is given in the following pages. One of the topics of that discourse was the absence of atmosphere from the Moon. This was accounted for by the kinetic theory of gas; inasmuch as the potential of gravitation on the Moon is such that a free molecule moving in any outward direction with a velocity of 2.38 kilometers[4] per second

[1] Reprinted from an advance copy of a paper in the *Transactions of the Royal Dublin Society*, Vol. VI, Part 13. Communicated by the author.

[2] See an extract from this Memoir on p. 28, below.

[3] According to the Kinetic theory Dalton's Law will be true of mixtures of gases if the free paths of the molecules between their encounters are straight. This is the case, to an excessively close approximation, in all laboratory experiments; but the law ceases to hold at elevations in the atmosphere where the longer and more slowly pursued free paths are sensibly bent by gravity.

[4] It is very desirable that the names of metric measures should be made English words, and pronounced as such. Thus kilometer, hektometer, and dekameter should be pronounced with the accent on the second syllable, as in thermometer, barometer,

would escape, and, accordingly, the Moon is unable to retain any gas, the molecules of which can occasionally reach this speed at the highest temperature that prevails on the surface of the Moon.

Shortly after, a second communication was made to the Royal Dublin Society, at one of its evening scientific meetings, based on the supposition that the Moon would have had an atmosphere consisting of the same gases as those of the Earth's atmosphere, were it not for the drifting away of the molecules. It was shown that if the molecules of these gases can escape from the Moon, it necessarily follows that the Earth is incompetent to imprison free hydrogen ; and this was offered as explaining the fact that, though hydrogen is being supplied in small quantities to the Earth's atmosphere by submarine volcanoes and in other ways, it has not, even after the lapse of geological ages, accumulated in the atmosphere to any sensible extent. This communication was followed at intervals by others, in which the investigation was extended to other bodies in the solar system, in which an endeavor was made to trace what becomes of the molecules that filter away from these several bodies, and in which it was suggested that the gap in the series of terrestrial elements between hydrogen and lithium may be accounted for by the intermediate elements (except helium) having escaped from the Earth at a remote time, when the Earth was hot.

In one of the earlier of these communications, it was pointed out that it is probable that no water can remain on Mars — a probability which is now raised to a certainty by the recent discovery, that helium (with a molecular mass twice that of hydrogen) is being constantly supplied in small quantities to the Earth's atmosphere by hot springs, and probably in other ways, and that nevertheless there is no sensible accumulation of it in the Earth's atmosphere after the infiltration has been going on for cosmical ages of time. In the absence of water, carbon dioxide was sug-

etc. This would have the further useful effect of better distinguishing these names from decimeter, centimeter, and millimeter, which have accents on the first and third syllables.

gested as, with some probability, the substance that produces the polar snows upon Mars. Moreover, on the Earth, snow, rain, and cloud are produced by the lightest constituent of our atmosphere ; but if the atmosphere of Mars consist of nitrogen and carbon dioxide, snow, frost, and fog on that planet are being produced by the heaviest constituent. An attempt was made to follow out the consequences of this state of things, and to refer to it those recurring appearances upon Mars which, though very imperfectly seen owing to the great distance from which we observe them, have been (perhaps too definitely) mapped and described under the name of canals.

Of this series of communications, though known to many, only imperfect printed accounts have appeared ; and it is the object of the present communication to present the subject in a more complete form. The opportunity will be taken of substituting better numerical results for those originally given, by basing them on the fact which has recently come to our knowledge, that not only hydrogen, but helium also, with a density twice that of hydrogen, can escape from the Earth. The most notable change that this makes is, that what was before probable is now certain — that water cannot in any of its forms, be present upon Mars.

CHAPTER I.

Of the fundamental facts.—In order to see why neither hydrogen nor helium remains in the Earth's atmosphere, and why there is neither air nor water on the Moon, it is necessary to understand the conditions which determine the limit of an atmosphere. These were investigated under the kinetic theory of gas by the present writer in a memoir communicated to the Royal Society in May, 1867 : see his paper "On the Physical Constitution of the Sun and Stars," in the *Proceedings of the Royal Society*, No. 105, 1868, from page 13 of which it will be convenient to make the following extract : [1]

[1] Further information on this subject will be found in sections 22, 24, 25, 26, and in the footnote to section 93, of the paper here quoted.

"23. Let us consider what it is that puts a limit to the atmosphere. Let us first suppose that it consists of but one gas, and let us conceive a layer of this gas between two horizontal surfaces of indefinite extent, so close that the interval between them is small compared with the mean distance to which molecules dart between their collisions, but yet thick enough to have, at any moment, several molecules within it. Molecules are constantly flying in all directions across this thin stratum. Some of them come within the sphere of one another's influence while within the layer, and therefore pass out of it with altered direction and speed. Let us call them the molecules emitted by the layer. If the same density and pressure prevail above and below the layer, the molecules which strike down into it will, on account of gravity, arrive with somewhat more speed on the average than those which rise into it. Hence those molecules which suffer collision within the stratum will not scatter equally in all directions, but will have a preponderating downward motion, so that of the molecules emitted by the stratum more will pass downwards than upwards. This state of things is unstable, and will not arrive at an equilibrium until either the density or the temperature is greater on the underside of the layer. If the density be greater, more molecules will fly into the stratum from beneath than from above ; and if the temperature be greater the molecules will strike up into it, both more frequently and with greater speed. In the Earth's atmosphere it is by a combination of both these that the equilibrium is maintained ; both the temperature and the density decrease from the surface of the Earth upwards."

"24. We have hitherto taken into account only those molecules which, after a collision, have arrived at the stratum from the side on which the collision took place. But besides these there will be a certain number of molecules which, having passed through the stratum from beneath, fall back into it without having met with other molecules, either by reason of the nearly horizontal direction of their motion, or because of their low speed. The number of molecules that will thus fall back into the stratum will be a very inconsiderable proportion of the whole number passing through the stratum, so long as the temperature and density are at all like what they are at the surface of the Earth. In the lower strata of the atmosphere, therefore, the law by which the temperature and density decrease will not be appreciably affected by molecules thus falling back. But in those regions where the atmosphere is both cold and very attenuated, where accordingly the distance between the molecules is great and the speed with which they move feeble, the number of cases in which ascending molecules become descending without having encountered others will begin to be sensible. From this point upwards the density of the atmosphere will decrease by a much more rapid law, which will, within a short space, bring the atmosphere to an end."

It appears, then, that the atmosphere round any planet or

satellite will, *cæteris paribus*, range to a greater height the less gravity upon that body is ; and that if the potential of gravitation be sufficiently low, and the speed with which the molecules dart about sufficiently great, individual molecules will stream away from that body, and become independent wanderers throughout space.

Thus, we shall presently see that, in the case of the Earth, a velocity of about eleven kilometers per second (nearly seven miles) would be enough to carry a molecule at the boundary of our atmosphere off into space, if the Earth were alone and at rest ; and a somewhat less velocity of projection (about 10.5^{km} per second) is sufficient, on account of the rotation of the Earth, and because westerly winds sometimes blow in the upper regions of the atmosphere. The modification introduced by these subsidiary causes will be examined in Chapter IV, and the amount of their effect will be determined. The behavior of molecules is also slightly affected by the Moon, which is near enough sensibly to alter the orbits of molecules if shot up in some directions.

Let us now consider what would happen if free hydrogen could remain in our atmosphere. Hydrogen is, in modern times, being supplied in small quantities to the Earth's atmosphere by submarine[1] volcanoes and in other ways. Even if there were no tendency in hydrogen to leak away, it could not, in the free state become a *large* constituent of our atmosphere, because, when it came to be a certain proportion of the atmosphere, it would, on the occasion of the first thunderstorm or on account of fires, enter into combination with the oxygen, which is, in modern times, a large constituent of the atmosphere ; but after each such explosion it would accumulate until it became a minor constituent like carbon dioxide were it not for the events described in this paper ; and in former times, before there was vegetation to evolve free oxygen, it might have been a large constituent but for those events. The free hydrogen which con-

[1] The hydrogen evolved by terrestrial volcanoes burns into water on reaching the air, and ceases to be free hydrogen.

tinues in modern times to be supplied in small quantities to the atmosphere is used up in some way. A little may be occluded, some may suffer surface condensation, and the rest is escaping.

The evidence that there is an escape of gas from the Earth's atmosphere is still more conspicuous in the case of helium. Small quantities of this gas are constantly being dribbled into the atmosphere by hot springs and probably in other ways, and it was probably supplied more copiously in former times. Now helium is so little disposed to enter into combination with other elements, that the efforts of chemists to effect any such union have been unavailing. We must conclude, therefore, that this gas remains unchanged within the atmosphere, where it would therefore, in the lapse of time, have accumulated so as to be now a sensible and perhaps a large constituent of the Earth's atmosphere were it not that it is escaping from the atmosphere's outer boundary as rapidly as it enters it below—indeed so promptly escaping, that the amount *in transitu* is too small for the appliances of the chemist to detect it.

On the other hand, water is not sensibly leaving the Earth. From which we learn that the potential of the Earth and the temperature at the boundary of its atmosphere are such as enable our planet effectually to imprison the vapor of water with molecules whose mass compared with molecules of hydrogen is 9 (and probably ammonia with a density of 8.5.). The other constituents of the Earth's atmosphere, such as nitrogen, oxygen, and carbon dioxide, have still heavier molecules. Accordingly, none of these escape in sufficient numbers to produce any perceptible diminution of the quantity of gas upon the Earth.[1] We may infer from this that *the boundary between those gases that can effectually escape from the Earth and those which cannot, lies somewhere between gas consisting of molecules with twice the*

[1] We need not suppose that there is absolutely no escape of the molecules of the denser gases, but only that the event is an excessively rare one. Thus, if the molecules of a gas escape so very seldom that only a million succeed in leaving the entire atmosphere of the Earth in each second, then a simple computation will show that it would take rather more than 30 millions of years for a uno-twenty-one (the number represented by 1 with 21 ciphers after it) of these molecules to have escaped. Now a

mass of molecules of hydrogen and gas with molecules whose mass is nine times[1] *the mass of molecules of hydrogen.*

This we may take to be one fact which we can ascertain by observing what occurs upon the Earth, and the telescope has been able to reveal to us another fact of a like kind, viz., that there is either no atmosphere upon the Moon, or excessively little—a fact which has been made certain by the application of very delicate tests.

CHAPTER II.

Interpretation by the kinetic theory.—In order to make these facts the starting-point for fresh advances, we must study their precise physical meaning when interpreted by the kinetic theory of gas.

The velocity whose square is the mean of the squares of the velocities of the individual molecules of a gas—"the velocity of mean square" as it has been called—was determined[2] by Clausius to be

$$w = 485\sqrt{\frac{T}{273\,\sigma}} \text{ meters per second,}$$

where w is the velocity of mean square, T the absolute temperature of the gas measured in Centigrade degrees, and σ its specific gravity compared with air. We shall find it convenient to use ρ instead of σ, where ρ is the density of the gas compared with hydrogen. Accordingly $\sigma = \rho / 14.4$, whereby Clausius' formula becomes

uno-twenty-one is about the number of molecules which are present within every cubic centimeter of the gas at such temperatures and pressures as prevail at the bottom of our atmosphere. An escape of molecules of the denser constituents of the atmosphere on this excessively small scale, or even on a scale considerably larger, may be and probably is going on. See a paper on the "Internal Motions of Gases" in the *Philosophical Magazine* for August 1868, where the number of molecules in a gas is estimated. Readers of that paper are requested to correct a mistake at the end of the third paragraph, where 16^2 was by an oversight inserted instead of $\sqrt{16}$.

[1] We shall find in the chapter on Venus that the presence of water on that planet enables us to somewhat lower the upper of these two limits.

[2] *Phil. Mag.*, **14**, 124, 1857.

$$w = (111.4)\sqrt{\frac{T}{\rho}}, \qquad (1)$$

in meters per second. This formula gives a velocity of 1603 meters, nearly a mile a second as the "velocity of mean square" in hydrogen at an absolute temperature of 207°, *i. e.*, at a temperature which is 66° C. below freezing point. This is the "velocity of mean square" of the molecules of hydrogen in an atmosphere consisting either wholly or partly of hydrogen, at any situation in which the gas is at that low temperature. Similarly by putting $\rho = 2$ and $T = 207$, we find the velocity of mean square for helium at the same low temperature. It is about 1133 meters per second. The actual velocities of the molecules are, of course, some of them considerably more and others considerably less than this mean, even if the hydrogen or helium be unmixed with other gases; and the divergences of some of the individual velocities from the mean will become exaggerated when the encounters to which the molecules of these lighter gases are subjected are sometimes with molecules many times more massive, and which may, when the encounter takes place, be moving with more than their average speed, as must often happen in our atmosphere. Under these circumstances we should be prepared to find that a velocity several times the foregoing mean is not unfrequently reached; and the evidence (see Chapter IV) goes to show that *a velocity which is between nine and ten times the velocity of mean square*, a velocity which is able to carry molecules of either hydrogen or helium away from the Earth, *is sufficiently often attained to make the escape of gas effectual.*

We are now in a position to aim at making our results so definite that they may be extended to other bodies in the solar system.

CHAPTER III.

Dynamical equations.— In making our calculations with reference to the planets and satellites of the solar system, it will simplify the work, and be sufficient for our purpose, to treat

them as spherical bodies, consisting of layers each of which is a spherical shell of uniform density. In that case, if B be one of these bodies

a (the acceleration of the surface of B, due to attraction)

$$=\frac{M}{R^2}, \tag{2}$$

and

$$K \text{ (the potential of gravitation at its surface) } =\frac{M}{R}, \tag{3}$$

where M is the mass of B, and R the radius of its spherical surface.

Now K, the potential, as we learn in the science of dynamics, expresses the kinetic energy stored up per unit of mass by a small[1] body in falling upon the surface of B from infinity. Hence,

$$K=\frac{v^2}{2}, \tag{4}$$

where v is the velocity which would be acquired by a small mass

[1] By a small body is to be understood one whose mass bears to the mass of B, a ratio so small that, from the physical standpoint, it may legitimately be regarded as a small quantity of at least the first order. For this purpose, a ratio of a tenthet, that is, of a unit in the tenth place of decimals, is sufficiently small in almost every branch of physical inquiry. If M be the mass of B, and m the mass of the body falling upon it, then the energy changed from potential into kinetic energy, by allowing them to fall together from infinity,

$$=\frac{m\,v^2}{2}+\frac{M\,V^2}{2},$$

if we suppose them to have started from rest, and if on coming together they have acquired the velocities V and $-v$. Now, by the principle of the center of mass, $MV+mv=0$. Therefore the acquired kinetic energy may be written

$$=\frac{m\,v^2}{2}\left[1+\left(\frac{m}{M}\right)\right],$$

which differs from being

$$=\frac{m\,v^2}{2}$$

by an insensible quantity if the ratio m/M is sufficiently small. And it is much more than sufficiently small from the physical standpoint, in the cases we are concerned with, where m is the mass of a gaseous molecule, and M the mass of a planet or satellite. In fact m is here of about the fifth order of small quantities compared with M, if we take a tenthet (10^{-10}) as about the ratio between quantities of two consecutive orders.

in falling from infinity. If a missile were projected from B with this speed, it would just be able to reach infinity, *i. e.*, this speed is the least which would enable a molecule to get completely away from B. We may, therefore, call it *the minimum speed of escape* from B when B is at rest. If B rotates, a less velocity relatively to the surface of B will suffice, provided that the missile is shot off in the direction towards which the station from which it starts was being carried by the rotation at the instant of projection.

CHAPTER IV.

Of the Earth.—Let us apply these elementary dynamical considerations to the Earth. In doing this, we may assume—

R (the Earth's equatorial radius), - -	$= 6378$ kilometers
h (the height of the atmosphere), - -	$= 200$ "
g (gravity at E, a station on the equator, at the bottom of the atmosphere), - -	$= 978.1^{\text{cm}}./\text{sec.}/\text{sec.}$
u (the velocity at the equator due to the Earth's rotation), - - - -	$= 464^{\text{m}}./\text{sec.}$

We shall need one other datum, viz., the highest temperature which can be reached by the air at station E', where E' is a station at the top of the atmosphere, over the equator. To enable us to arrive at definite results, we shall regard this temperature as -66° C. Our numerical results would be affected, but would only suffer a slight alteration, by substituting for this particular temperature any other which is admissible. It is, accordingly, legitimate to make our computation on this assumption, viz., that the temperature at station E' is 66° C. below freezing point. At this temperature Clausius' formula, equation (1) gives for the velocity of least square in a gas

$$w = (111\cdot4)\sqrt{\frac{207}{\rho}},$$

$$= 1603 / \sqrt{\rho}, \qquad (5)$$

if we here use w to signify the velocity of least square at this particular temperature.

Let us next calculate a, the acceleration due to the attraction of the Earth at station E (on the equator, and at the bottom of the atmosphere). Here

$$a = g + \gamma, \tag{6}$$

where g is gravity at the equator, and γ the acceleration due to the Earth's rotation, *i. e.*,

$$\gamma = \frac{u^2}{R} = \frac{(464\ \mathrm{II})^2}{6378\ \mathrm{V}},$$
$$= 3.4^{\mathrm{cm.}}/\mathrm{sec.}/\mathrm{sec.}, \tag{7}$$

where we use II for the two additional ciphers, and V for the five additional ciphers, which are necessary to express u and R in C. G. S. measure.[1]

Introducing this value of γ into equation (6), we find

$$a = g + \gamma = 978.1 + 3.4$$
$$= 981.5\ \mathrm{cm.}/\mathrm{sec.}/\mathrm{sec.} \tag{8}$$

Again, $a = M/R^2$, where M, the mass of the earth, is expressed in gravitation units, and K' (the potential at station E', which is at the top of the atmosphere) $= M/(R + h)$. Taking the ratio of these we get rid of M, so that it is immaterial in what units it has been expressed. We thus find

$$K' = a\frac{R^2}{R + h} = (981{\cdot}5)\frac{(6378\ \mathrm{V})^2}{6578\ \mathrm{V}},$$
$$= l^{-1}\ 11.7830, \tag{9}$$

where l^{-1} means "the number whose logarithm is." This result is expressed in C. G. S. measure.

Now $K' = v^2/2$ (see equation 4), where v' is the minimum speed of projection which would carry a molecule clear away from the Earth, if the Earth were stationary. We thus find

$$v' = 1101500\ \mathrm{cm.}/\mathrm{sec.},$$

[1] The author has found it very convenient, especially in investigations touching on molecular physics, to use Roman figures to represent factors consisting of 1 followed by the number of ciphers indicated by the Roman figure. In this way VI means a million, XII means a billion; similarly, XXI means a uno-twenty-one, which is about the number of gaseous molecules in each cubic centimeter of air at the bottom of our atmosphere.

which is the same as

$$v' = 11.015 \text{ km./sec.} \tag{10}$$

Now the rotation of the Earth carries station E' along at the rate of 0.478 km./sec. Hence a velocity

$$\begin{aligned} v' - u' &= 11.015 - 0.478, \\ &= 10.537 \text{ km./sec.,} \end{aligned}$$

will suffice, if the molecule be shot off in the direction in which it is already traveling in consequence of rotation. And, finally, if a strong west wind is blowing at station E', which must sometimes happen, a speed of

$$v' - u' - a = 10.5 \text{ km./sec.,} \tag{11}$$

may be enough. This, then, we may take to be the least velocity which enables molecules to escape from the Earth.

Let us now turn to what happens in gas. By Clausius' formula, p. 31,

$$w \text{ (the velocity of mean square in a gas)} = (111 \cdot 4)\sqrt{\frac{T}{\rho}} \text{ m./sec.,}$$

which, at 66° below zero (which we regard as the temperature at station E') gives

$$\begin{aligned} w &= (111 \cdot 4)\sqrt{\frac{207}{\rho}}, \\ &= 1603 / \sqrt{\rho} \text{ m./sec.,} \end{aligned} \tag{12}$$

where w means the velocity of mean square in a gas at the temperature — 66° C.

If in this we put $\rho = 1$, we find

$$w = 1603 \text{ m./sec. in hydrogen.}$$

This is nearly a mile a second. Similarly putting $\rho = 2$, we find

$$w = 1133 \text{ m./sec. in helium.}$$

which is somewhat more than a kilometer per second. And, finally, if we put $\rho = 9$, we find

$$w = 534 \text{ m/sec., in the vapor of water,}$$

which is somewhat more than half a kilometer per second.

Now, we found above that, in order that any gas may cease to be imprisoned by the Earth, its molecules must now and then

be able to attain at least a speed of 10.5 kilometers per second; see equation (11). Whenever this happens to a molecule favorably circumstanced it escapes. Hence, since hydrogen succeeds in leaking away from the Earth, its molecules must in sufficient numbers attain this speed, which is 6.55 times the velocity of mean square in that gas at a temperature 66° below zero; and since helium can escape, its molecules must sufficiently often reach a speed equal to or exceeding 9.27 times what we have found to be the velocity of mean square in helium at a temperature of —66° C.

On the other hand, in order that a molecule of water may escape from the Earth, it has to get up a speed of 19.66, nearly twenty times the velocity of mean square in that vapor at the above temperature: and the fact that water does not drain away from the Earth in sensible quantities shows that this seldom happens.

We are now in a position to make a very important deduction in molecular physics from these facts, which is that *in a gas a molecular speed of 9.27 times the velocity of mean square is reached sufficiently often to have a marked effect upon the progress of events in nature;* while, on the other hand, a molecular speed of twenty times the velocity of mean square is an event which occurs so seldom that it exercises no appreciable influence over the cosmical phenomena which we have been considering. We must remember, however, that there are other events in nature—in chemistry, and especially in biology—which may be, and probably are, determined by conditions that occur far more rarely.

The separation of the swiftest moving molecules from the boundary of our atmosphere is, of necessity, accompanied by a lowering *pro tanto* of the temperature of the atmosphere left behind. It is one of the many operations carried on by nature to which the second law of thermodynamics does not apply. We must remember that this law is only a law of molecular averages, and therefore is not a law of nature where, as in this case, nature separates one class of molecules (those moving fastest) from the rest.

CHAPTER V.

Extension of the inquiry to other bodies.—In order to extend our inquiry to the atmospheres upon other bodies of the solar system, we have to determine the potential of gravitation upon them. We can do this where r, the radius of the new body B, and m/M, the ratio of its mass to the mass of the Earth, are known. For then

$$k \text{ (the potential at the surface of } B) = \frac{m}{r}$$

$$= \frac{m}{M} \cdot \frac{R+h}{r} \cdot \frac{M}{R+h}, \tag{13}$$

of which the last factor is the K' which is given in a numerical form in equation (9).

Combining this with the dynamical equation (see p. 33)

$$k = v^2/2 \tag{14}$$

we can calculate v, which would be the minimum velocity of escape from B, if B were at rest. In general B rotates, and then the minimum velocity of escape is

$$v' = v - u, \tag{15}$$

where u, the velocity at the equator of B due to its rotation, is easily found, if we know from observation the period of rotation.

Having calculated v', we can determine what density a gas must have to escape from B with the same facility with which helium leaves the Earth. For this purpose, let w_1 be its velocity of mean square. Then, in accordance with what is stated on p. 37, w_1 may be as large as

$$w_1 = \frac{v'}{9{\cdot}27}, \tag{16}$$

where w_1 and v' are to be expressed in meters per second: and then Clausius's equation, viz.

$$w_1 = (111{\cdot}4)\sqrt{\frac{T}{\rho_1}} \text{ m./sec.}, \tag{17}$$

enables us to calculate ρ_1/T, *i.e.*, the density of that gas which, at a specified temperature T, can escape from B as freely as

helium does from the Earth at a temperature of $-66°$ C. This and all lighter gases will escape.

To determine what density of gas will be imprisoned by B as firmly as water is by the Earth, we proceed in a similar way. Here

$$w_2 = \frac{v'}{19 \cdot 66}, \tag{18}$$

and the rest of the work is the same as before, giving as its result the value of ρ_2 / T, where ρ_2 is the density of a gas which will find it as difficult to escape from B as water does from the Earth. It and all denser gases will be retained.

The investigation leaves uncertain the fate of gases whose density lies between ρ_1 and ρ_2.

CHAPTER VI.

Of the Moon.—When we turn to the Moon, we find the conditions to be such that it can rid itself of an atmosphere with much ease. Upon the Moon

r (its radius), ... $= 1738$ km.

$\frac{m}{M}$ (the ratio of its mass to that of the Earth), . $= 0.01235$

P (its period of rotation) . . . . $= 2,360,591$ sec.

Calculating v', the least velocity which would enable a missile to quit the Moon by the equations in the last chapter, we find it to be about 2.38 km./sec., while on the Earth it is 11.015 km./sec., which, by the help of the rotation of the Earth and possible storm, may be, under favorable circumstances, furnished by a relative projectile velocity of 10.5 km./sec. Accordingly, more massive molecules can disengage themselves from the Moon with the same facility with which helium can leave the Earth, if ρ, their molecular mass, is greater than that of helium, in the ratio of the square of 10.5 to the square of 2.38, *i. e.*, if the molecules are 19.5 times heavier than those of helium, or, which is the same thing, 39 times heavier than those of hydrogen. Accordingly, hydrogen sulphide, with a molec-

ular mass 17 times that of hydrogen, oxygen with a molecular mass of 16, nitrogen with a molecular mass of 14, and the vapor of water with a molecular mass of 9, will hurry away. They will all escape with greater facility than hydrogen does from the Earth. A like fate will befall argon with a molecular mass of 20, carbon dioxide with its molecular mass of 22, carbon disulphide with its molecular mass of 38, and all others of the gases emitted by volcanoes, or from fissures, of which the vapor density is less than 39. These will escape with greater promptness than does helium from the Earth.

This is what would happen if the Moon were by itself, and if portions of its surface could rise even to a temperature of — 66°C. But the conditions are more favorable. Lord Rosse infers from his observations that the temperature of the Moon's surface rises something like 280°C. when exposed to the fierce glare of the Sun's rays. Even if it shall turn out that this is an overestimate, it at all events makes it probable that the maximum temperature is very much higher than 207° above the absolute zero, which is the same as 66° below the freezing point. Moreover, the proximity of the Earth would somewhat assist the process at its present distance ; and its greater proximity in former ages must have more assisted it. In fact, on this account, any of the gases or vapors in question which had been developed upon the Moon while the Moon was close to the Earth must have been for the most part transferred over to the Earth, if the Earth was then cool enough to retain them. Those molecules that have escaped from the Moon since its distance from the Earth became considerable have for the most part become independent planets traveling in a ring round the Sun, of which ring (roughly speaking) the Earth's path is the central line. There they are accompanied by most of the molecules of hydrogen and helium that have leaked away from the Earth. A very few of the latter which happened to be shot off at unusually high speed, and in the direction towards which the Earth was at the time traveling in its orbit, may have been able to disengage themselves altogether from the solar system ; but

this can have happened to but few of those thrown off from the Earth, and not to almost any of those ejected from the Moon.

CHAPTER VII.

Of Mercury.—The radius of Mercury may be obtained by assuming the equatorial radius of the Earth to be 6378 kilometers, and applying to it the data given in the preface to the *Nautical Almanac* for 1899. We thus find the planet's radius

$$r = \frac{3''{\cdot}34}{8''{\cdot}848} 6378 = 2406 \text{ km.}$$

The mass of Mercury is less satisfactorily known. We shall use the value

$$\frac{m}{M} = 0{\cdot}065.$$

Mercury's rotation period is also in doubt. The difficult observations that have hitherto been made seem to be about equally consistent with a rotation period of nearly a day, and a rotation period of 88 days (the period of Mercury's revolution round the Sun). Possibly observations could be made in the daytime which would determine between these. Meanwhile

$u = 2$ m. / sec., if the rotation period is 88 days.
$u = 175$ m. / sec., if the rotation period is 1 day.

By using the above values for r and m / M in equations (13) and (14), we find

v (the minimum velocity of escape, if Mercury were at rest) = 4643 m. / sec.,

which is a little more than 4½ km. / sec. Hence

$v' = v - u = 4641$ m. / sec., if the rotation period is 88 days,

and

$= 4468$ m. / sec., if the rotation period is 1 day.

By employing these values in equations (16) and (17), we find that

ρ (the density of the gas that will escape from Mercury, as freely as helium does from the Earth) . . = 10.25, on the 88-day hypothesis

and = 11, on the 1-day hypothesis,

and on the further supposition that the absolute temperature of the gas where it escapes is 207°, that is 66° C. below zero.

If the highest temperature at the upper surface of Mercury's atmosphere over his equator is higher than this, and it is probably much higher, the foregoing values for ρ will have to be increased in the ratio of $T / 207$, where T is the highest temperature reached. It must also be remembered that helium is so prompt in escaping from the Earth that it is probable that gases somewhat denser could escape ; and, as a consequence, that the limiting density of the gases that can escape from Mercury has to be increased in the same proportion.

The general conclusion then is :

1. That water with a density of 9 certainly cannot exist upon Mercury. Its molecules would very promptly fly away.

2. That it is in some degree probable that both nitrogen and oxygen, with densities of 14 and 16, would more gradually escape.

It is, therefore, not likely that there are, in whatever atmosphere Mercury may be able to retain, any of the constituents of the Earth's atmosphere except perhaps argon and carbon dioxide.

CHAPTER VIII.

Of Venus.—The state of Venus' atmosphere need not detain us long. The potential of gravitation is so nearly the same on this planet as on the Earth that its atmosphere almost certainly retains and dismisses the same gases as does the atmosphere of the Earth. The only element of uncertainty arises from its period of rotation being imperfectly known, but the nearly globular form of the planet assures us that its rotation cannot be swift enough seriously to affect the problem.

The similarity of the two atmospheres is confirmed by the appearance of the planet. Venus is presumably a much younger planet than the Earth, and its temperature is consequently what the Earth's was many ages ago, when through excessive evaporation water was the largest constituent of our atmos-

phere, and when clouds were present everywhere and without intermission.

The conditions upon Venus are so nearly akin to those on the Earth that we cannot be mistaken in regarding the vapor which forms the abundant cloud we see on that planet as none other than the vapor of water. If we may assume this, we can advance a step farther than the statements made in Chapter IV.

The detailed computations in the case of Venus give

$$r = \frac{8''.40}{8''.848} 6378 = 6053 \text{ kilometers,}$$

$$\frac{m}{M} = 0{\cdot}769;$$

and as such observations as are practicable seem to indicate that on that planet

$$P = 83779 \text{ seconds,}$$

we find that

$$v = 10000 \text{ m. / sec.,}$$
$$u = 454 \text{ m. / sec.;}$$

whence we infer that

$$v' = v - u = 9546 \text{ m. / sec.,}$$

is the least speed which will carry a projectile away from Venus.

Now, in water, $\rho = 9$. Whence, in accordance with Clausius' formula, p. 31, the velocity of mean square in water, at the temperature of $-66°$ C., is

$$w = \frac{1603}{\sqrt{\rho}} = 534 \text{ m. / sec.}$$

Now v' is almost exactly 18 times this value of w; so that the circumstance that Venus is able to retain its hold upon water means that the molecules of a gas do not attain a velocity 18 times that of mean square sufficiently often to enable the gas to escape from an atmosphere in appreciable quantities.

We are accordingly now in a position to go beyond the statement made on p. 314. We may now say:

1. A velocity of 9.27 times that of mean square is attained by the molecules of a gas sufficiently often to enable helium to escape from the Earth.

2. A velocity 18 times that of mean square is so seldom attained that Venus has been able to retain its stock of water.

3. Since Venus can prevent the escape of water, the Earth, with its larger potential, is competent to retain its hold upon a gas of somewhat less density, viz., one whose density is $\rho = 7.43$.

Accordingly, as regards the Earth, we may come to the following conclusions: (1) Gases with a density of 2 or less than 2 can certainly escape from the Earth; (2) a gas with a density of 7.43, and all denser gases,[1] are effectually imprisoned by the Earth; (3) the information supplied by Venus, supplemented by our present chemical knowledge, does not determine what would be the fate of a gas, if there be such, whose density lies between 2 and 7.43.

CHAPTER IX.

Of Mars.—The case of Mars is one of exceptional interest. Using the data furnished by the *Nautical Almanac*, we find its radius to be

$$r = 3372^{\mathrm{km}}.$$

As in the case of Mercury, its mass is not yet known with exactness. It has become better known since observations have been made on the elongations of its satellites, which seem to furnish the value:

$$\frac{m}{M} = 0{\cdot}1074.$$

Its period of rotation is known, viz., 88,643 seconds; whence, and from its radius, we find

$$u \text{ (the velocity at the equator due to rotation)} = 239^{\mathrm{m}}/\mathrm{sec}.$$

By following the same steps as in the case of Mercury, we find successively

$$v = 5042^{\mathrm{m}}/\mathrm{sec.},$$

[1] Ammonia NH_3, and Methane CH_4, are a little above this limit, and therefore can neither of them escape. Ammonia is no doubt washed out of the Earth's atmosphere by rain; but it is not easy to see what becomes of the methane. It seems unlikely on chemical grounds that it directly combines with oxygen, furnishing water and carbon dioxide. Possibly it meets with a trace of chlorine, and furnishes methyl chloride and hydrogen in the presence of sunshine; or possibly it is nitro-methane that is formed.

for the least velocity which would carry a missile away from Mars, if Mars were not rotating, and

$$v' = v - u = 4803^{m}/\text{sec.},$$

for the relative velocity which is sufficient in consequence of the rotation.

From this, and equations (16) and (17), we find

$$\rho = 9.57,$$

as the density of a gas which would escape from Mars at a temperature of -66° C., with the same facility as helium from the Earth. Hence, and since $9.57 : 9 = 207 : 194.7$, it follows that water would quit Mars at the absolute temperature of 194.7°, that is at -78.3° C., as freely as helium can escape from the Earth at the temperature of -66° C.

We must make some allowance for the probability that the highest temperature at which a gas has an opportunity of escaping from Mars may be lower than the corresponding temperature on the Earth. And we must, on the other hand, remember that the molecules of helium are almost certainly not quite the heaviest molecules that can rid themselves of the Earth. Taking both considerations into account, *it is legitimate to infer that water, in which $\rho = 9$, cannot remain on Mars.*

As to what happens to gases with densities of 14 and 16, we cannot speak with confidence. They may perhaps be imprisoned. And the conspicuous polar snows of Mars make it in a considerable degree probable that carbon dioxide, of which $\rho = 22$, is abundantly present.

It appears here to be worth reviewing the state of things that must prevail if the atmosphere of Mars consists mainly of nitrogen and carbon dioxide. Without water, there can be no vegetation upon Mars, at least not such vegetation as we know; and, in the absence of vegetation, it is not likely that there is much free oxygen. Under these circumstances, the analogy of the Earth suggests that the atmosphere of Mars consists mainly of nitrogen, argon, and carbon dioxide.

Carbon dioxide, the most condensible gas of such an atmos-

phere, would behave very differently from the way in which water behaves on the Earth. Water in the state of vapor is so much lighter than the other constituents of our atmosphere that it hastens upward through the atmosphere; and, accordingly, its condensation into cloud, whether of droplets of water or spicules of ice, takes place usually at very sensible elevations. There would be no such hurry to rise on the part of carbon dioxide, it would, on the contrary, show great sluggishness in diffusing upward through an atmosphere of nitrogen. When brought to the ground in the form of snow or frost (for there would probably be no rain), and when subsequently evaporated, the carbon dioxide gas would crawl along the surface, descending into valleys, occupying plains and pushing its way under the nitrogen, mixing only slowly with the nitrogen; and, as a result, only a very small proportion of the whole stock would be at any one time found elsewhere in the atmosphere than near the ground. It is suggested that the fogs, the snows, the frosts, and the evaporation of such a constituent of the atmosphere may account for the peculiar and varying appearances upon Mars, which, though recorded in our maps as if they were definite, are in reality very imperfectly seen from our distant Earth. In fact, Mars, when nearest the Earth, which unfortunately seldom happens, is still 140 times farther off than the Moon. Fogs over the low-lying plains which on Mars correspond to the bed of our ocean, with mountain chains projecting through the fog, and a border of frost along either flank of these ranges, would perhaps account for some of the appearances which have been glimpsed; and extensive displacements of the vapor, consequent upon its distillation towards the two poles alternately, would perhaps account for the rest.

CHAPTER X.

Of Jupiter.—In the case of the planet Jupiter, we have the following data:

$$r \text{ (Jupiter's equatorial radius)} = \frac{97''\cdot 36}{8''\cdot 848} 6378 = 70170 \text{ km},$$

P (the periodic time of his rotation) $= 35{,}728$ seconds,

$\frac{m}{M}$ (m being Jupiter's mass, and M the mass of the Earth) $= 311{\cdot}9$.

Using these data we find—

u (the velocity at his equator, owing to the rotation) $= 12.337^{\text{km}}/\text{sec.}$,

v (the least velocity which would carry a missile away, if Jupiter were not rotating) $= 59.570^{\text{km}}/\text{sec.}$,

$v' = v - u$ (the least velocity which enables a missile to escape when helped by the rotation) $= 47.233^{\text{km}}/\text{sec.}$,

ρ_1 (the density of gas which would escape from Jupiter, at a temperature of $-66°$ C., with as much ease as helium does from the Earth) $= 0.099$ of the density of hydrogen,

ρ_2 (the density of a gas which would be imprisoned by Jupiter as effectually as water is by Venus) $= 0.373$ of the density of hydrogen.

Hence gases with a density less than $\frac{1}{10}$ of that of hydrogen (if any such exist) could escape from Jupiter. But Jupiter can prevent the escape of a gas which has a density a little more than a third of the density of hydrogen, and of all denser gases.

Jupiter is accordingly able to imprison all gases known to chemists. His atmosphere may therefore, so far as can be determined by the present inquiry, have in it all the constituents of the Earth's atmosphere, with the addition of helium and hydrogen, and any elements between hydrogen and lithium which the Earth may have lost; except that, if the hydrogen is sufficiently abundant, there can be no free oxygen. Owing to the chemical reaction that would then take place, the oxygen will have been used up in adding to the stock of water.

CHAPTER XI.

Of Saturn, Uranus, and Neptune.—Our information with reference to these three planets is less satisfactory. Computing their radii from the data given in the *Nautical Almanac*, we find

$r = 61060$ km. on Saturn,
$= 24700$ km. on Uranus,
$= 26340$ km. on Neptune.

Their masses compared with the masses of the Earth are also sufficiently known, viz.:

$$
\begin{aligned}
m / M &= 93.328, \text{ for Saturn,} \\
&= 14.460, \text{ for Uranus,} \\
&= 16.863, \text{ for Neptune;}
\end{aligned}
$$

but their rotation periods are very imperfectly known. We shall take them to be about

$$
\begin{aligned}
P &= 36864 \text{ seconds, of Saturn,} \\
&= 36000 \text{ seconds, of Uranus,} \\
&= 36000 \text{ seconds, of Neptune.}
\end{aligned}
$$

If we may use these values, we find

$$
\begin{aligned}
u &= 10.412 \text{ km./sec., on Saturn,} \\
&= 4.311 \text{ km./sec., on Uranus,} \\
&= 4.598 \text{ km./sec., on Neptune,}
\end{aligned}
$$

for the velocity at the equator due to the planet's rotation. Further, by equations (13) and (14), we find for the minimum velocity of escape from each of these planets, if not rotating,

$$
\begin{aligned}
v &= 34.92 \text{ km./sec., on Saturn,} \\
&= 21.61 \text{ km./sec., on Uranus,} \\
&= 22.60 \text{ km./sec., on Neptune;}
\end{aligned}
$$

whence

$$
\begin{aligned}
v' = v - u &= 24.508, \text{ on Saturn,} \\
&= 17.299, \text{ on Uranus,} \\
&= 18.002, \text{ on Neptune,}
\end{aligned}
$$

is the least velocity which enables a missile to escape when helped by the rotation.

By dividing these last numbers by 9.27, we find the velocity of mean square of the gas which can escape as freely as does helium from the Earth, and then by Clausius' formula, we can calculate ρ_1, its density, which is

$$
\begin{aligned}
\rho_1 &= 0.37 \text{ of the density of hydrogen on Saturn,} \\
&= 0.74 \text{ of the density of hydrogen on Uranus,} \\
&= 0.68 \text{ of the density of hydrogen on Neptune.}
\end{aligned}
$$

On the other hand, by dividing the values for v' by 18, we

learn what is the velocity of mean square of the gas which would be detained as firmly as water is held by Venus; and then, if we calculate ρ_2 by Clausius' formula, we find

$\rho_2 =$ 1.39 times the density of hydrogen on Saturn.
$=$ 2.78 times the density of hydrogen on Uranus.
$=$ 2.57 times the density of hydrogen on Neptune.

Now hydrogen, with a density of 1, stands in each case between ρ_1 and ρ_2, and we are, therefore, left uninformed whether hydrogen is or is not allowed to escape. There is, perhaps, some ground for conjecturing that it cannot escape from Saturn, and that it can escape from Uranus and Neptune. But this must remain doubtful. Helium, with its density of 2, being more than the value of ρ_2 upon Saturn, is certainly imprisoned by that planet, but we have no satisfactory information as to what is its fate upon Uranus or Neptune.

Thus the information we gain with reference to these three planets amounts to this—that we have no definite information as regards hydrogen; that Saturn is able to detain helium, but that we do not know whether the other two planets can or cannot; that all other gases known to chemists would be more firmly imprisoned by any one of these planets than they are by the Earth; and that, if there be gases lighter than hydrogen, it is certain that Saturn cannot detain any of which the density falls as low as one-third of that of hydrogen, Neptune cannot hold any as light as two-thirds, nor Uranus any lighter than three-quarters of the density of hydrogen. On the whole, the probability seems to be that the atmosphere of Saturn is nearly the same as that of Jupiter; while the atmospheres of Uranus and Neptune more nearly approximate to that of the Earth, with perhaps the addition of any gases with densities less than 7.43 that may possibly have left the Earth when the Earth was hotter, and whose withdrawal from the Earth is perhaps what has left the gaps in the series of terrestrial elements which appear to exist between hydrogen and helium, and between helium and lithium.

CHAPTER XII.

Of the satellites and minor planets.—We have no sufficient information as to the densities of any these bodies. But the asteroids, or minor planets, which lie between the orbits of Mars and Jupiter, are all of them bodies so small that, even if they were as dense as osmium, iridium, or platinum, they could not retain their hold upon an atmosphere. The same may be said of the two satellites of Mars, of the two satellites of Jupiter, of most of the satellites of Saturn, and of the small bodies that make up the rings of Saturn. None of these can condense any atmosphere upon them. If there are molecules of gases traveling in their neighborhood, they also are, each of them, an independent satellite.

One satellite of Saturn and three of Jupiter are larger than our Moon; and one other of Saturn and one of Jupiter, though smaller than the Moon, are not much smaller. We should need to know the densities of these bodies before we could speak with confidence about them. The presumption, however, is that, as their primaries are very much less dense than the Earth, so these satellites are probably less dense than the Moon. If so, they also, as well as the smaller satellites, must be devoid of atmosphere.

We know too little about the satellites of Uranus and Neptune to venture upon any conclusion about them. The satellite of Neptune appears to be a body of considerable size, and, with some probability, it may have an atmosphere.

CHAPTER XIII.

What becomes of the molecules that escape.—The speed of the Earth in its orbit is about 30 km./sec. Now it follows, from the dynamics of potential, that the potential of the Sun at the distance of the Earth is represented by the square of this number if the Sun's mass be measured in gravitational units. That is

$$k = \frac{m}{r} = 900,$$

where m is the mass of the Sun, and r the radius of the Earth's orbit.

We have already found, on p. 35, the potential of the Earth at the boundary of our atmosphere to be

$$K' = \frac{M}{R + h} = \frac{v'^2}{2} = \frac{121}{2} = 60{\cdot}5.$$

Therefore the joint potential of the Sun and Earth at that station is

$$\frac{m}{r} + \frac{M}{R + h} = 960{\cdot}5.$$

This, then, is equal to $v^2/2$, when v is the least velocity which would enable a missile to escape from both these bodies if stationary. Therefore

$$v = \sqrt{(2 \times 960{\cdot}5)} = 43{\cdot}83 \text{ km.} / \text{sec.}$$

If the missile be shot off in the direction towards which the Earth is traveling, it has already got, in common with the rest of the Earth, 30 km./sec. of this velocity; and therefore, if fired off in that direction, the speed with which it would need to part from the Earth is 13.83 km./sec. Accordingly, this is a velocity which would suffice to set the molecule completely free, if the Earth were arrested in its orbit immediately after the molecule left it. But since, on the contrary, the Earth persists on its course, a slightly greater speed of projection is actually needed. Now, as 11 km./sec. is enough to enable a molecule to leave our atmosphere, it can be but very seldom that a molecule quits it with a velocity somewhat exceeding 13.83 km./sec.; and, accordingly, nearly all the molecules that have left the Earth have remained in the solar system, and are in fact now traveling as independent planets round the Sun.

We have taken the special case of a molecule leaving the Earth's atmosphere. A similar treatment applies to molecules leaving the atmospheres of other planets and satellities. In every case the velocity required to enable a molecule to quit the solar system is markedly in excess of that which enables it to escape

from its own atmosphere. Accordingly, almost all such wandering molecules are still denizens of the solar system.

CHAPTER XIV.

Former size of the Sun.—The Sun is contracting, and therefore in past time was larger than it now is. The question then arises, how much larger may it have been while it was still globular? We can place a limit on its possible size *if we assume that it was then, as now, able to prevent the escape of free hydrogen*, and if we assign a temperature below which its outer boundary did not fall.

In order to arrive at definite results, let us suppose this temperature to be 0°C. Here we might take into consideration the probability that, at a sufficiently remote period, the planets formed part of the Sun. But it is needless to do this, as the addition to be then made to its present mass would be only about $\frac{1}{750}$ part, which is too slight an increase sensibly to affect our present computation.

We have first to ascertain what the "velocity of mean square" of hydroden is at the freezing temperature. It is got by putting $T = 273$ and $\rho = 1$ into Clausius' formula, page 310. We thus find $w = 1.841$ km./sec. This multiplied by 9.27 (see page 314) gives us a velocity v_1 which the molecules of hydrogen could, at this temperature, get up sufficiently frequently, for the purpose of escape. And if multiplied by 18 (see page 320), it furnishes a velocity v_2 which hydrogen is unable to get up sufficiently frequently for effective escape. We thus find

$$v_1 = 17^{\text{km}}/\text{sec.} \qquad , \qquad v_2 = 33.14^{\text{ km}}/\text{sec.}$$

We have next to find how large the Sun should be in order that one or other of these velocities should be that which is just sufficient for the escape of a molecule. For that, r_1 and r_2 being the corresponding radii, the potentials must amount to

$$\frac{m}{r_1} = \frac{17^2}{2} = 144.5, \qquad , \qquad \frac{m}{r_2} = \frac{(33.14)^2}{2} = 549.$$

But at the distance of the Earth we found $m/r = 900$. Therefore

$$\frac{r_1}{r} = \frac{900}{144.5} = 6.227, \qquad \frac{r_2}{r} = \frac{900}{549} = 1.64.$$

That is, the surface of the Sun would need to have been about $6\frac{1}{4}$ times farther from the Sun than the Earth now is, in order that hydrogen at 0°C. should escape from it as freely as helium does from the Earth at —66° C. And it would need to have been 1.64 times farther than the Earth to imprison the hydrogen as firmly as water is held by Venus.

Hence, the *greatest* size which the Sun can have had since it became a sphere, consistently with its not allowing hydrogen at 0° C. to escape, is an immense globe extending to some situation intermediate between the orbits of Mars and Jupiter. From some such vast size it may have been ever since slowly contracting.

CHAPTER XV.

Of motions in a gas.—In carrying on an inquiry such as that of the present Memoir, we should keep in mind that the encounters between molecules have not the same effect on their subsequent motions as mere collisions between elastic or partially elastic solids would have. Let us, for simplicity, picture to ourselves two molecules which approach one another along a straight line, and after an encounter, which is in fact a complex struggle, recede from one another along the same line.

If they were solid particles with elasticity e, the equations of their motion would be

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2,$$
$$u_1 - u_2 + e(v_1 - v_2) = 0,$$

where $v_1 v_2$ are the velocities before, and $u_1 u_2$ the velocities after, the collision, and where e, the coefficient of elasticity, depends on the amount of the kinetic energy which is expended on internal events during the collision. It is therefore necessarily a proper fraction; so that e, in the case of solid particles, cannot exceed 1, whereas, in the encounters between molecules, it may have any value whether above or below 1. This is because, dur-

ing an encounter between molecules, energy is in some cases imparted to, and in other cases withdrawn from, the motions of the molecules along their free paths, whereas, in a mere collision, energy is always withdrawn. In fact, the *internal* events of individual molecules are in communication with heat motions in the ether, and interchange energy with it. A molecule may thus absorb energy from the ether during the whole of the long flights which it makes, when near the top of an atmosphere, between its encounters; and any excess of energy thus acquired will be shared with the motions of translation of the molecules when the next encounter takes place. Accordingly, the value of e will vary from one encounter to another, and, near the boundary of an atmosphere, there may be changes in the velocities of the molecules which are more abrupt than in situations where the gas is denser.

The effect here spoken of would be more marked in the case of helium, water, nitrogen, or oxygen, than in that of hydrogen, inasmuch as solar rays of the kind that hydrogen can absorb reach the Earth in a feebler state than those which the other gases absorb, owing to the partial absorption by hydrogen which has already taken place in the hot outer atmosphere of the Sun. On this account the rays that can affect hydrogen are the relatively feeble radiations from Fraunhofer lines, whereas the molecules of the other gases are exposed on the confines of our atmosphere to the glare of full sunshine. This is evidenced by the Earth-lines of the solar spectrum, especially those due to oxygen and aqueous vapor.

These considerations were taken into account in fixing on -66° C. as the maximum temperature to be attributed to the outer layer of our atmosphere. No doubt it would, in some slight degree, improve the investigation to use a rather lower temperature in the solitary case of hydrogen; but it was not thought necessary to make a distinction of this kind in an investigation which, from the nature of the case, could only be approximate. The only effect of introducing the refinement would have been to show that the facility with which hydrogen escapes

from an atmosphere is not quite so much in excess of the facility with which helium escapes as the numbers in Chapter IV indicate. This is almost certainly true to some small extent; but it leaves our main conclusions undisturbed. Accordingly, the simpler mode of inquiry, in which these and other small differences are ignored, has been an adequate investigation for our purpose.

HELIOGRAPHIC POSITIONS. III.

By Frank W. Very.

The last step in the passage from plane polar to heliographic coördinates shall now be explained. Fig. 4, Plate XXIV,[1] is a diagram representing the more important points required in the determination of heliographic positions, as they would be seen from the Earth at about the time of the autumnal equinox. P is the Sun's north pole. K and N' are the intersections of the Sun's surface with the axes of the ecliptic and of our celestial sphere, passing through the Sun's center. $ABMN$ is the solar equator, AB its invisible and BMN its visible branch, N being its ascending node. A line from the Sun's center to the Earth pierces the Sun's surface at C, the center of the visible solar disk, and the arc CM (symbol $= D$) measures the heliographic latitude of the Earth's position, or of the point on the Sun which has the Earth at its zenith. $N'C$ is a celestial hour-circle, and PD is a solar meridian through the spot (S) whose heliographic latitude is SD (symbol $= d$). The heliographic longitude of the Earth from the Sun's node is $NABM$ (symbol $= L$), and that of S is $NABD$ (symbol $= l$). Once in each solar rotation the assumed prime meridian coincides with the node. It will simplify matters to assume that the drawing is made at one of these recurrences of the conditions obtaining at the primary epoch, in which case the *nodal* heliographic longitude, $NABD$, will also be the heliographic longitude of the spot from the prime meridian, PN, or the condition $l' = l$ will be fulfilled.

To avoid complicating the figure, the ecliptic axis KC, and a line at right angles to it, through C, representing the ecliptic trace, also a line at right angles to $N'C$, through C, terminating in the points (E and W) of a celestial parallel, are omitted.

In the triangle PSC, the angle PCS is the algebraic difference of the position-angle of the Sun-spot ($P = N'CS$), reckoned

[1] *Ap. J.*, **6**, Dec., 1897.

from north towards east as a positive angle, and the position-angle of the Sun's north pole. The latter angle is $N'CP$, or in our symbols, $-(G+H)$, taking G and H with the signs already given. Hence

$$PCS = N'CS - N'CP = P + G + H = \chi.$$

The arc SC (symbol $= \rho$) is the angular distance of the spot from the center, and since $PC = 90^\circ - D$, the solution of the triangle PSC gives us:

$$\sin(90^\circ - PS) = \cos SC \sin(90^\circ - PC) + \sin SC \cos(90^\circ - PC) \cos PCS,$$

$$\sin SPC = \frac{\sin PCS \sin SC}{\sin PS},$$

or substituting the more general symbols:

$$\sin d = \cos\rho \sin D + \sin\rho \cos D \cos\chi.$$
$$\sin(L - l) = \sin\chi \sin\rho \sec d$$

which are the fundamental equations enabling us to state the heliographic latitude of a spot and its heliographic longitude from the Earth's position.

The spot's longitude from the solar prime meridian is determined as follows: Since there are no fixed reference points on the Sun's visible surface, we must assume an imaginary prime meridian, or, if this be possible, one determined by some recurring phenomenon connected with the solar rotation. If Carrington's prime meridian has been chosen, and if the time of the Sun-picture is stated in astronomical reckoning, which at present is taken to begin at mean noon, we must transform the interval from noon into the decimal of a day and look up the day of the year in the almanac, taking the beginning of the year, or $0^d.00$ at mean noon of December 31, of the preceding civil year. We then have the interval in days from Carrington's epoch

$$t = 365(y - 1854) + (d - \tfrac{1}{2}) + b,$$

where y is the year, d is the day of the year and fraction of a day counting from mean noon (or in present astronomical reckoning), and b is the number of leap years which have intervened between 1854 and the beginning of the current year. The latter

year, if it be a leap year, and the date after February 28, will have its extra day included in the day of the year as given in the almanac. *If civil time is used, d must be diminished by one, instead of by one-half.*

We must now divide by the sidereal period of the Sun's rotation, getting:

$$\frac{t}{25.38} = m + T,$$

where T is the remainder, or the interval of time from the previous conjunction of the solar prime meridian with the Sun's node. Converting this interval into degrees, we have for the nodal heliographical longitude of the prime meridian:

$$l' = 360° \times \frac{T}{25.38} = 14°.1844 \times T.$$

If the nodal heliographic longitude of the spot be l its true heliographic longitude will be $l - l'$, reckoned always from the prime meridian toward heliographical east, or in the same direction in which right ascensions and celestial longitude are reckoned.

Some observers, while using Carrington's system, have departed from his epoch, and employ the solar meridian which coincided with the Sun's node at Greenwich mean *noon*, Jan. 1, 1854, their epoch being exactly twelve hours later than Carrington's.[1]

[1] In the *Greenwich Spectroscopic Observations* for 1884, we read (p. 106): "The rotations adopted in the following table correspond to the synodic rotation of the Sun, and the commencement of each is defined by the coincidence of the assumed prime meridian with the central meridian, the assumed prime meridian being that meridian which passed through the ascending node at mean noon on January 1, 1854, and the assumed period of the Sun's sidereal rotation being 25.38 days. The rotations adopted in the volumes of *Greenwich Observations*, 1877 to 1883, correspond on the other hand to the sidereal rotation of the Sun, the commencement of each being defined by the coincidence of the assumed prime meridian with the ascending node. The numeration of the rotations is in continuation of Carrington's series (*Observations of solar spots made at Redhill* by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853, Nov. 9. The dates of commencement of the rotations are given in Greenwich civil time reckoning from midnight." The statement that the "numeration of the rotations is in continuation of Carrington's series" seems sufficiently explicit, but the execution of the precept is discordant. Apparently someone has erred in carrying the times forward, by failing to notice that the dates, or intervals, were given originally in civil time, a practice which is most convenient for daylight observations.

Consequently their heliographic longitudes will be $7^\circ\ 5'.5$ greater than those expressed in the original system. Carrington's statement is as follows: "As the fractions of the day [intervals] are throughout counted in civil time from the preceding midnight, 1854 — $0^d.000$ here signifies mean midnight on December 31, 1853." He says further that he has given, "(1) the day and fraction from midnight of the observation [expressed usually not as a date, but as an interval]; (2) the difference from the preceding epoch; (3) this difference converted into rotation angle in the proportion of $360^\circ : 25^d.38$,[1] or the angle through which the prime meridian had rotated since its last coincidence with the node. The deduction of this amount for each day manifestly leaves us a heliographical longitude reckoned in all cases from a prime meridian, which if our period be correct, is constant, if incorrect, varies slowly with the time." (*Observations of the Solar Spots*, p. 16.)

The solar prime meridian which coincided with the node, 1854, Jan. 1.000 (civil date), arrived at the center of the solar disk, 1854, Jan. 3.0057, a date determined by the condition

$$L - l' = 0.$$

The mean synodic period of the Sun's rotation is given by the equation

$$S = \frac{365.25636}{\frac{365.25636}{25.38} - 1} = 27^d.27523,$$

or the diurnal synodic rotation is on the average $13^\circ.1988$, or about $13^\circ 12'$; but since the Sun's longitude changes $4'$ faster on January 1 than on July 1, the synodic rotation varies in the course of the year, and owing to the day and year being incommensurable, we cannot draw up a table of the exact values of $L - l'$ according to the dates which will be true for

[1] The sidereal rotation $25^d.38$ was adopted because of "its admitting conveniently of much subdivision without remainders;" but it happens to be a close, as well as a convenient approximation for the phenomena of spot rotation.

all years.[1] The simplest method of computing the dates of conjunction of the solar prime meridian with the center is to assume an approximate date, compute L and l' for this date and apply to it the correction

$$\frac{L-l'}{14^\circ.1844 - \Delta\odot},$$

where $\Delta\odot$ is the diurnal change of the Sun's longitude on the given date, the unit of the correction being the mean solar day.

Carrington gives the date of his first published position

$$\text{Nov. } 9, 1853 = 1853, 312^{d}.489,$$

stating the time as an interval from the beginning of the civil year; that is to say, the observation was made at 0.489 of the 313th day of the year 1853, civil reckoning. Central conjunction of the prime meridian occurred on this same day, 1853, Nov. 9.3625, or $3^h.036$ before Carrington's observation. This constitutes the beginning of "Rotation No. 1," and the central conjunction of Jan. 3.0057, 1854, was the beginning of "No. 3." As an example of the computation, let us carry the last forward twenty years. We must add

$$268 \times 27.27523 = 7309.7616$$

to 3.0057, and subtract

$$(20 \times 365) + 5 = 7305. \text{ days},$$

there having been five leap years during the interval, giving Jan. 7.7673, 1874, as a first approximation to the date of the beginning of the 271st rotation. The correction to the first approximation is in this case $+0^d.0494$, and the date of the beginning of rotation No. 271 is January 7.8167, 1874.

A little care in regard to the epoch of the rotations will be necessary if the student of this subject would avoid vexatious

[1] SIR ROBERT S. BALL, in his *Atlas of Astronomy* (p. 21), has adopted a plan which will give approximate values. He tabulates a mean correction to $L - l'$, from which interpolated angles can be taken and subtracted from the value of $L - l'$ accurately computed for noon on the *previous* first day of January. It may be noted that Professor Ball repeats the error that "the moment of noon on January 1, 1854," "was the epoch selected by Carrington."

discrepancies. I will take an example from the *Companion to the Observatory* for 1889 (No. 144, p. 37). We find there the following data for January 1, 1889: $P = +1° 34'$, $D = -3° 19'$, $L = 133° 49'$; and this explanation is given:

The position-angle of the Sun's axis, P, is the position-angle of the north end of the axis from the north point of the Sun, read in the direction N., E., S., W. In computing D (the heliographic latitude of the center of the Sun's disk), the inclination of the Sun's axis to the ecliptic has been assumed to be 82° 45′, and the longitude of the ascending node to be 74°. In computing L (the heliographic longitude of the center of the disk)[1] the Sun's period of rotation has been assumed to be 25.38 days, and the meridian which passed through the ascending node at the epoch 1854.0 has been taken as the zero meridian.

This description of the epoch is not sufficiently explicit. We might question whether the year is the common year beginning January 0 in astronomical parlance, that is noon of December 31, 1853, or whether the civil year beginning at midnight between December 31, 1853, and January 1, 1854, is meant. Since Carrington is careful to state that the latter is the epoch chosen by him, the incautious student might be pardoned for supposing that it is the one adopted here, but a computation[2]

[1] Called $L - l'$ in this article.

[2] Illustration, using the nomenclature of this article, but epoch of *Companion*:

1889. January 1.5 (civil) $\odot = 281° 20'.6$ $\omega = 23° 27'.2$

$N = 74°$

" " " " $\odot - N = 207° 20'.6$ $I = 7° 15'$

$l \tan \omega$	9.63733	$l \tan I$	9.10454	$l \cos I$	9.99651
$l \cos \odot$	9.29378	$l \cos (\odot - N)$	9.94854n	$l \tan (\odot - N)$	9.71358
$l \tan G$	8.93111	$l \tan H$	9.05308n	$l \tan L$	9.71009
		$l \sin I$	9.10106		
		$l \sin (\odot - N)$	9.66211n		
		$l \sin D$	8.76317n		

$G = +4° 52'.6$ $H = -6° 26'.8$ $L = 27° 09'.4$ $D = -3° 19'.4$

Position-angle of Sun's north pole (referred to center of disk, and reckoned from N. towards E.) $= -(G + H) = +1° 34'.2$.

Interval from *mean noon*, January 1, 1854 = 12784 days.

$503 \times 25.38 = 12766.14$ "

$T = 17.86$ "

$17.86 \times 14°.1844 = l' = 253° 20'.0$

$L = 387\ \ 9\ .4$

Heliographic longitude of center $= L - l' = 133° 49'.4$

shows that neither the first mentioned nor the second year is used, but one beginning at Greenwich mean noon on January 1, 1854.

Carrington appears to have considered his determination of N trustworthy to certainly less than a degree. Taking his exact value ($73° 40'$ for 1850) and allowing for precession, the value of N for 1889 is $74° 12'.6$, which, with Carrington's epoch, gives for midnight 1889, January 1. 0 (civil), the interval being the same in both cases,

$$\odot = 280° 50'.0 \qquad L = 26° 26'.4$$
$$\odot - N = 206° 37'.4 \qquad D = -3° 14'.5$$
$$L - l' = 133° 6'.4.$$

The change of epoch chiefly affects the longitudes, and these may be readily transformed from one system to the other. For example, since the prime meridian revolves with the Sun's sidereal rotation, while the center of the disk revolves according to the Sun's synodic rotation,

Companion's value, $L - l' = 133° 49'.4$, January 1, 1889, noon,
minus ½ true daily $\odot$ rotation, $-7° 5'.5$,
minus difference in assumed N, $-12'.6$,
gives Carrington's value, $L - l' = 126° 31'.3$, January 1, 1889, noon.
This *plus* ½ synodic daily $\odot$ rotation, $+6° 35'.1$,
gives Carrington's value $L - l' = 133° 6'.4$, for previous midnight.

An example of the complete reduction of a Sun-spot position is appended. The spot in question was a large one in rapid change, and the leading one of a great group. The position of the apparent center of the particular umbra chosen, altered rapidly from day to day, and the original observation, being made on a small paper drawing is possibly inaccurate to a considerable fraction of a degree. Nevertheless, it will serve to illustrate the method. The drawing, which is reproduced in Plate I, was made by Mr. F. Slocum, a student at this Observatory, by the method of projection, using the 12-inch equatorial with its aper-

ture reduced to six inches.[1] Meridians and parallels 30° apart, have also been drawn, giving a very effective idea of the solar

[1] Spot *A*. September 18, 1896. See Mr. Slocum's note, p. 92 of this number.

Date = 1896, September 18.33 Greenwich M. T. (astronomical),
= 1896, " 18.83 " " " (civil),
= 0.83 of the 262d Greenwich day of the year 1896,
= 15601^d.83 from 1854.0 (civil).

Radius of spot, corrected for distortion = $\frac{r'}{R'} = \frac{1.85}{3.00} + 0.016 = 0.633$.

$P = 302°\ 35'$. $R'' = 958''.2$.

Reduction from tabular $\odot = +1161''$.

$\odot = 176°\ 2'\ 59'' + 19'\ 21'' = 176°\ 22'\ 20''$.

$N_{1850} = 73°\ 40'$ }
Precession to 1896.717 $= +39'\ 9''$ } $74\ 19\ 9$

$\odot - N = 102°\ 3'\ 11''$
$\omega = 23°\ 27'\ 17''$
$I = 7°\ 15'\ 0''$

$l \tan \omega$	9.63736	$l \tan I$	9.10454	$l \cos I$	9.99651
$l \cos \odot$	9.99913n	$l \cos (\odot - N)$	9.31977n	$l \tan (\odot - N)$	0.67055n
$l \tan G$	9.63649n	$l \tan H$	8.42431n	$l \tan L$	0.66706n

$l \sin I$ 9.10106
$l \sin (\odot - N)$ 9.09032
$l \sin D$ 9.09138

$G = -23°\ 24'\ 46''$, $D = +7°\ 5'\ 22''$, $H = -1°\ 31'\ 18''$
$L = -77°\ 51'\ 9'' = 282°\ 8'\ 51''$

$\rho' = 0.633 \times 958.2 = 607''$ $P = 302°\ 35'$
$\rho = \sin^{-1}(0.633) - 607''$ $(G + H) = -\ 24°\ 56'$
$= 39°\ 16' - 10' = 39°\ 6'$ $\chi = 277°\ 39'$

$l \cos \chi$	9.12425			$l \sin \chi$	9.99612n
$l \cos D$	9.99667	$l \sin D$	9.09138	$l \sin \rho$	9.79981
$l \sin \rho$	9.79981	$l \cos \rho$	9.88989	$l \sec d$	0.00708
	8.92073		8.98127	$l \sin (L - l)$	9.80301n

$\sin d = 0.08332 + 0.09578 = 0.17910$
$= \sin 10°\ 19'.0$

Interval from 1854.0 = 15601^d.83
614 × 25.38 = 15583 .32
Remainder = T = 18^d.51

$L - l = -39°\ 26'.7$
$L = 282\ \ 8\ .9$
$l = 321\ \ 35\ .6$
$18.51 \times 14°.1844 = l' = 262\ \ 33\ .2$
$L - l' = 59°\ 2'.4$

Spot *A*, heliographic latitude $= +10°\ 19'$, heliographic longitude $= 59°\ 2'$, referred to the initial epoch of Carrington, or if the epoch be taken twelve hours later, heliographic longitude $= 66°\ 8'$.

presentation. Since the computation of heliographic positions has to be repeated many times, it is desirable to employ a printed form; and since the values of I and N are not known with quite the degree of accuracy implied in the accompanying example, the labor may be further abridged by tabulating the values of G, H, L, and D, for single degrees of the arguments, $\odot$ and $\odot - N$.

Heliographic latitudes can be determined somewhat more accurately than longitudes, because less affected by foreshortening near the limb. They are also subject to less actual variation, because the large vertical movements in solar spots are combined with the normal axial rotation of the Sun's mass, which is sufficient to give a drift in the direction of the latitude parallels. The longitudinal elongation of Sun-spot groups bears witness to this tendency.

Spots close sometimes by horizontal inrush, but more frequently by alterations which are probably to be interpreted as a physical change of state, occurring over a wide area, rather than as extensive transportation of materials. But however produced, the result is a sudden shifting of apparent spot-positions.

As a favorable example of spot-recurrence, and an illustration of the amount of variation which may be expected in spot-positions, I select the following groups observed by Carrington (see table on opposite page). The columns headed Δl and Δd exhibit the apparent diurnal changes in heliographic longitude and latitude. A positive drift in latitude means one towards the pole.

It will be seen by a comparison of positions that the groups are really but a single one which, on further comparison, proves to be nearly identical with one of the previous March, a recrudescence which is not unusual. A new number, however, has been given to the group at each reappearance by the Sun's rotation. The positions bracketed together are those of individual spots belonging to the group, which fluctuate and disappear; but the initial spot was exceptionally long-lived. Its mean

No.	Date and interval	Heliographic longitude	Δl	Heliographic south latitude	Δd
	1860				
710	May 4	22° 13′	?	11° 27′	?
	124.496	12 23	?	11 53	?
710	May 5	21 56	— 17′	11 43	+ 16′
	125.492	12 26	+ 3	11 30	— 23
710	May 6	21 40	— 15	11 17	— 25
	126.553	———			
710	May 7	21 27	— 14	11 35	+ 19
	127.485	15 48	?	12 58	?
710	May 9	21 28	+ 1	11 2	— 15
	129.644	13 23	?	11 55	?
710	May 13	21 2	— 7	11 32	+ 8
	133.627	———			
730	May 30	———			
	150.382	21 23	+ 1	12 36	+ 4
730	June 5	30 31	?	16 14	?
	156.358	21 16	?	18 50	?
		20 10	— 12	12 29	— 1
730	June 6	29 15	— 64	15 58	— 13
	157.546	19 39	— 26	12 47	+ 15
730	June 8	32 37	?	15 2	?
	159.528	24 31	?	17 56	?
		19 6	— 17	12 48	+ 1
753	June 25	18 5	?	21 21	?
	176.616	18 22	— 3	13 4	+ 1
753	June 26	20 38	?	20 37	?
	177.336	19 54	+ 128	12 46	— 25
753	July 1	27 14	?	16 31	?
	182.576	19 3	— 10	12 15	— 6
		14 56	?	18 45	?
		9 23	?	19 27	?
753	July 3	18 34	— 15	12 31	+ 8
	184.563	15 16	+ 10	18 58	+ 7
		9 22	— 1	19 22	— 3
753	July 4	18 32	— 2	12 26	— 5
	185.530	14 57	— 20	18 51	— 7
		8 0	— 85	18 52	— 30
753	July 6	14 29	— 13	18 47	— 2
	187.723	18 11	— 10	12 13	— 6
777	July 22	———			
	203.490	34 8	?	15 42	?
777	July 24	33 1	— 31	15 42	0
	205.629	22 37	?	16 51	?
		18 52	+ 2	11 41	— 2
777	July 25	33 16	+ 15	15 39	— 3
	206.641	23 3	+ 26	17 25	+ 34
		18 44	— 8	11 27	— 14
777	July 30	33 32	+ 3	14 47	— 11
	211.545	22 37	— 5	17 32	+ 1
777	Aug. 1	33 31	— 1	14 21	— 12
	213.664	22 14	— 11	17 0	— 15

positions and mean diurnal drifts, from one apparition to the next, were:[1]

	Interval from Jan 1.0	Long.	Δ l	South Lat.	Δ d
1st rotation	$126^d.6$	$21°.8$		$11°.5$	
2d rotation	156 .0	20 .1	−3′	12 .7	+2′
3d rotation	183 .5	18 .9	−2′	12 .4	−1′
4th rotation	206 .0	18 .8	0	11 .6	−2′

These and other similar mean variations being for long intervals, and representing the determinations of four to six days in each case, we may conclude that proper motions, or variations in the mean drift, of 3′ or 4′ per day, corresponding to horizontal linear movements of 370 and 493 miles per day, are established beyond a doubt. The larger and irregular fluctuations noted from day to day are in many cases also probably real, but since the movement is not continuous in any given direction, these fluctuations are eliminated from the mean. The mean deviations from the adopted period refer in this case to a spot in heliographic latitude 12° south, not far from the latitude (about 14°) which gives the average or typical solar rotation. The residuals in longitude are much larger than these as the latitude departs from that of the mean spot-zone, becoming about —40′ in latitude 35°. This constitutes the evidence for Carrington's great discovery that the solar surface does not revolve as one piece. We must distinguish, however, between those mean residuals which result from the combination of the periods of a large number of spots by the elimination of the average period, and those departures from the mean rotation for a given latitude, which indicate a proper motion or divergence from a mean surface drift. The latter are especially liable to occur in large and vigorous spots, and are of peculiar interest. The group selected for analysis was a comparatively quiet one, with small proper motion; but individual spots show occasional movements of large amount, and certainly much greater than the probable error of a single observation. In addition to the visible movements of spots whose unbroken history gives them an appear-

[1] *Observations of the Spots on the Sun*, p. 182.

ance of individuality and continuity which is doubtless far from the reality, we see the frequent birth of new spots in the rear of old ones, testifying, in all probability, to an invisible lateral transportation of materials at a very rapid rate. The spectroscope assures us that the lighter constituents of the Sun are moving in the higher regions of the chromosphere at very great velocities, and although we do not know whether the erupted masses of hydrogen, helium, calcium, and "coronium," are sufficient to account for spot growth, the intimate association of prominences and Sun-spots warrants the hypothesis of a chain of causal connection through a hidden, complex circulation of material between neighboring spots.

The study of proper motions either of a spot as a whole, or of its component filaments, plumes, bridges, or reticulations, needs extensive series of instantaneous photographs of the Sun, taken at short intervals on a large scale, and carefully reduced. Nothing comes amiss here. Bits of information given by spectroscopic measures of velocities in the line of sight, studies of facular development by the spectroheliograph, or of local variation of radiative power by the bolometer, observations of those evanescent changes in spot-forms and spectral details, which must escape detection by occasional photographic registration, and which can only be captured by continuous visual study, and the rarer events of coronal and chromospheric displays, must be combined together and interpreted by the aid of extensions of chemical and physical theory suitable to the novel problem to be dealt with, before we can have anything like a complete life-history of a Sun-spot. Evidently coöperation is needed, and the fragmentary contributions of individual workers are not to be despised.

In finishing a Sun-drawing, there is a temptation to bring any group of spots which may be under inspection near the margin, to the center of the field, in order to secure the advantage of a sharper image. But such a procedure must be executed with great caution, lest the relative positions of details be altered because of the difference in distortion. If the leading positions

and outlines have been drawn to scale with a correctly centered image, there is no objection to a free-hand infilling of the finer detail seen under better conditions, but alterations of scale will vitiate any conclusions which may be drawn as to positions or dimensions.

The determination of spot-dimensions is intimately connected with that of positions. With a transparent plate of glass or mica, ruled in squares (*e. g.*, 0.5^{mm} on a side), apparent areas of umbras or penumbras may be easily estimated. These areas require correction for distortion and foreshortening near the limb. The area of one of the little squares at the center of the Sun's disk in terms of the area of the visible solar hemisphere, will be $\frac{1}{2\pi R^2}$, R being here the number of divisions of the counting plate in the radius of the projected solar disk; while near the limb, the measured areas must be increased in the ratio, $\sec\rho : 1$; and the general expression for the area of the Sun's spherical surface, covered by a square division, is: $\frac{\sec\rho}{2\pi R^2}$, ρ being the heliocentric angle of the position from the center of the disk. If optical distortion has increased the marginal radius from r' to r, the measured area (a) must be diminished, for this cause, in the proportion, $1 : \left(1 + \frac{r - r'}{r'}\right)^2$; whence the final value of the corrected spherical area is:

$$a' = \frac{a \sec\rho}{2\pi R^2}\left(\frac{r'}{r}\right)^2.$$

This formula of course assumes that the area measured is a relatively small part of the sphere.

The summation of spot-areas has been pursued assiduously in the belief that it furnishes a reliable indication of solar surface activity. The methods followed in obtaining spot-areas are therefore important. In his memoir on *Photographs and Drawings of the Sun*,[1] Rev. S. J. Perry endeavors to account for a persistent difference in penumbral areas measured from photo-

[1] S. J. PERRY, *Mem. R. A. S.*, **49**, 286, 1889.

graphs and from drawings (the photographic areas being the larger) by suggesting that "from the state of the sky, or from the length of the exposure, or from a slight variation in the sensitiveness of the film, or from some modification in the development, the penumbra of a spot does not present so well defined an outline in the photograph as in the projected image, and that the measure is taken of what would appear to be the extreme limit of the spot."

Two other alternatives might be suggested. The first is that the drawings may have been finished with the spot removed to the center of the field in order to diminish chromatic aberration, thereby decreasing the size of the details drawn, even though the general disposition of the outline may not have been intentionally changed. Mr. Perry states, however, that "the unexplained differences for spots within 10° of the limb are almost nil, and those between 10° and 20° are exceptionally small.[1] This hypothesis, therefore, does not appear to be warranted in Mr. Perry's case.

My other suggestion is that the discrepancy is possibly sufficiently explained by the fact that the rays which affect the photographic plate are more strongly absorbed by the solar atmosphere than those which produce vision. It must always be a matter of judgment to decide how faint a shade shall be considered penumbral. The greater intensity of absorption of the photographic rays will throw the balance in favor of counting in a photograph what would be rejected in making a drawing. In favor of this hypothesis, it may be noted that very near the limb, the general absorption of the photographic rays is so great that variations of absorption, due to slight differences of penumbral elevation, may be masked, and the discrepancy between drawings and photographs diminished. The point is one which can probably be settled by comparing measurements on ordinary and on orthochromatic photographic plates.

LADD OBSERVATORY,
Providence, R. I., Aug. 1897.

[1] *Loc. cit.* p. 285.

ON THE CONDITIONS OF MAXIMUM EFFICIENCY IN ASTROPHOTOGRAPHIC WORK.

PART II. EFFECT OF ATMOSPHERIC ABERRATION ON THE INTENSITY OF TELESCOPIC IMAGES.

By F. L. O. WADSWORTH.

Introductory note.—When Part I of this paper was written it was intended to follow it at once with Part II (thus completing the general theory of contrast), and then apply the general results obtained to the consideration of the special cases, which have been referred to in detail elsewhere.[1] But as stated in another note,[2] this investigation had to be laid aside for a time to complete other work of a more pressing character, and it was therefore decided to publish, without waiting for the completion of the general theory, the results that had already been obtained in considering the special cases of the photographic and visual observations of the planets. In each of these cases (as in that of the objective spectroscope previously considered[3]) the effect of atmospheric aberration had been individually investigated.[4] There was, however, one other case which was considered at the same time, in which this effect was unfortunately not taken into account, not indeed because it was overlooked (any more than in the other cases), but because its importance in this particular line of work had been at first underestimated. This was in part due to the fact that we were concerned principally with small linear and angular apertures (the aberrational effect due to a given disturbance will vary in general, at least as the square, if not as a higher power of the aperture), and in part to there hav-

[1] See *Ap. J.*, **6**, 135; also *M. N.*, **57**, 588–589.

[2] See p. 77 of this number.

[3] *Ap. J.*, **3**, 62–64, June 1896.

[4] See paper "On the Photography of Planetary Surfaces," *Obs'y* **20**, 303, 365, 404, Nov. 1897. "On the Effect of the Size of an Objective on the Visibility of Linear Markings on the Planets." *Ast. Jour.*, 413, Oct. 1897.

ing been an error in one of the results used in developing the general theory of contrast for this case, which happened to be of such a nature and magnitude as to just mask the effect of atmospheric aberration, and thus enable conclusions to be reached which explained and were completely confirmed by the actual results of observation; a test of theory which is generally regarded as one of the most conclusive and satisfactory that can be applied. As it turns out, therefore, the main conclusions reached in this case are (through the balancing of the two errors, or rather one error and one omission) correct, if the general theory of contrast and "delineating power" upon which they are based is correct.[1]

The result which is referred to as in error, is the expression for I^2_{III} which represents the intensity of illumination of the focal plane due to an infinitely extended area. It does not enter into the expression for contrast in any, save the last, of the cases, yet considered; but it has been judged of sufficient theoretical importance at least to be considered, together with another error of an exactly similar character in the expression I^2_{II}, for *the intensity in the image of a long line*, more at length in another note.[2]

Taking into account these errors, the expressions for the contrast between the image and field in cases A, B, C, of the preceding paper become

$$\text{(A)} \qquad K_A - 1 = \frac{k_a}{k_{\text{III}}} \cdot \frac{\pi b^2}{4\lambda^2} = \text{Const.} \frac{k_a}{k_{\text{III}}} b^2 \qquad \text{(36a)}$$

[1] For further remarks on this point see pp. 82, 85 of this number.

[2] See p. 77 of this number. Wholesale criticism of the results of my recent investigations have been recently published by Professor Schaeberle (*Ast. Jour.*, No. 421) and Mr. Newall (*M. N.*, Nov. 1897) on the ground that this one expression I^2_{III} was *probably* in error. The cursory and superficial manner in which these gentlemen must have examined my papers is evidenced not only by this, but by the further facts, that both entirely overlook the references to Rayleigh's work; both fail to find the real error that was committed in his work and in my own (Professor Schaeberle's only analytical criticism is directed against a part of the work that is entirely correct.), and finally, as might have been expected, both entirely overlook the other error of exactly the same nature in the expression I^2_{II}. These facts have been pointed out and some other criticisms briefly answered in notes to the *Ast. Jour.*, No. 424, and to the *M. N.* (to be published probably in the January number), the publications in which Professor Schaeberle's and Mr. Newall's articles appeared.

$$(B)\ \begin{Bmatrix}(1)\ B_1\\(2)\ B_2\end{Bmatrix}\quad K_B - 1 = \frac{k_b}{k_{III}}\cdot\frac{8}{3}\cdot\frac{b}{\pi\lambda} \cong \frac{k_b}{k_{III}}\cdot\frac{1}{a} \qquad (37a)=(38a)=(40)$$

$$(C)\qquad K_C - 1 = \frac{k_c}{k_{III}} \qquad (39a)$$

The expression for case D is, as already given

$$K_D - 1 \cong \frac{k_d}{k_{III}}\cdot\frac{1}{a} \qquad (40)$$

i. e., the same as for B.

These expressions must be regarded as representing the effective contrast under:

1. Theoretically perfect conditions; *i.e.*, entire absence of both instrumental and atmospheric disturbances; or

2. Such conditions of observation as enable the effect of a moderate amount of atmospheric disturbance to be eliminated; *i. e.*,

a. Visual observations (with instruments of moderate aperture) in which as already pointed out[1] the eye makes the most of the intervals of "good seeing" without being influenced to any great extent by the intervening intervals of "fuzziness," due to temporary instrumental or atmospheric disturbances.

b. Photographic exposures which are either practically "instantaneous" as in the case of solar photography, or of the "intermittent" nature recently proposed by the writer[2] in which the favorable conditions of visual observations are, as nearly as may be, approximated.

The expressions for the *effective photographic contrast* in the case of prolonged continuous exposures will be developed in the present paper.

§ I. GENERAL CONSIDERATIONS.

It has long been recognized that the limits of the practical resolving and defining power of our astronomical instruments are set, not so much by instrumental limitations as to size and accuracy of workmanship, as by the difficulty of securing sufficiently

[1] *Pop. Astron.*, **5**, 205; *Obs'y* **20**, 336, Sept. 1897. [2] *Ibid.*

good atmospheric and meteorological conditions to enable the full theoretical advantages attendant upon increase of aperture to be practically realized. Long before the invention of the modern refractor, when the size of the largest instruments was counted in inches rather than feet as can be done now, this difficulty was clearly recognized by Newton,[1] and nearly every astronomer of note since his time has contributed something of either a theoretical or an experimental nature toward the determination of the most favorable meterological conditions, or the selection of the most suitable localities on the Earth's surface for various classes of astronomical and astrophysical observations. During the last half of this century, since instruments of large aperture have been rapidly coming into general use, many expeditions have been undertaken with the latter object particularly in view, and as a consequence a number of observatories of either a temporary or permanent character have been established at points which have been found to offer characteristic advantages for the prosecution of particular lines of work.

Among the more important of these expeditions may be mentioned that of Bond to the mountains of Switzerland in 1851; of Lassell to Malta in 1852; of Piazzi Smyth to Teneriffe in 1856; of the United States coast survey parties under Young and Davidson to the Sierras and the mountains of Wyoming in 1872; of Draper to the mountains of Utah and Wyoming in 1876; of Tacchini to Ætna in 1877; of Burnham to Mt. Hamilton in 1879; of Langley to Mt. Whitney in 1881; of Copeland to the Andes of Peru in 1883; of Abney to the Riffel in 1886; of Vallot and Janssen to Mt. Blanc in 1887–1890; of the Harvard College Observatory expeditions to Mt. Wilson and Peru in 1887–1890; of Hale to Pike's Peak and Ætna in 1893–4;

[1] "If the theory of making telescopes could at length be fully brought into practice, yet there would be certain bounds beyond which telescopes could not perform. For the air through which we look upon the stars is in a perpetual tremor, as may be seen by the tremulous motion of shadows cast from high towers and by the twinkling of the fixed stars. The only remedy is a most serene and quiet air, *such as may perhaps be found on the tops of the highest mountains above the grosser clouds*."—*Optice*, 107 (2d ed. 1719). See, also, Clerke's *History of Astronomy*, 519, and Holden's *Mountain Observatories*.

and of Mr. Lowell's expedition to Flagstaff, Arizona, and Mexico in 1894–1897.[1]

The subject of the general nature of atmospheric disturbances and their effect on the "seeing" has also received much attention. Arago, Laplace, von Humboldt, Montigny, Wolf, Hann, Secchi, Respighi, Dufour, Helmholtz, Davidson, Lord Rayleigh, André and Angot, Ventosa, Douglass, Hale, and many others have made important contributions of both an experimental and theoretical nature to our knowledge of the subject.[2] Much, however, still remains to be done in this field, particularly in the nature of systematic explorations of the meteorological conditions in the upper regions of the atmosphere, not by means of mountain stations, at which the conditions are abnormal because of the presence of the mountain itself, but over extensive plateaus and table lands by mean of self-registering instruments carried up by kites or captive balloons. Happily this field of work is now beginning to receive the attention it deserves.

In the present paper it is not so much our purpose to enter into a discussion of the cause and nature of atmospheric disturbances, as it is to inquire as to their effect on the *intensity* of the images formed at the focal plane of the telescope, a part of the subject that has not, as far as I know, been quantitatively and systematically investigated. In order to do this, it is, first of all, desirable to classify the effects of atmospheric disturbances on the position and character of the focal plane images.

[1] The attitude and expressed opinions of the Flagstaff observers in reference to this matter of the study of atmospheric conditions are curiously narrow ones. They believe evidently that theirs is a case in which "the last shall be first." Thus, according to Mr. Douglass (*Pop. Astron.*, **5**, 65, June 1897), "*Professor Pickering* (W. H.) *was unquestionably the first to intelligently appreciate the importance of seeking a good atmosphere*," and according to Dr. See (*A. N.*, No. 3438, 81), "*comparatively few observers have heretofore made a critical study of the conditions essential to good seeing* . . . . and . . . . *few observatories have been located for securing good definition and steady images*." In view of the actual facts of the case comment on such remarks seems unnecessary.

[2] For a very well written (though somewhat incomplete) review of the subject reference may be made to Holden's recent pamphlet, *Mountain Observatories*, published by the Smithsonian Institution; *Smith. Miss. Coll.*, No. 1035, 1896.

The various observed effects may be divided into three general classes :

1. Movements of the image as a whole without *sensible* loss of definition. These movements may be either
   a) Rapid and irregular ("jumping" or "dancing" of the image).
   b) Slow and oscillatory.
2. A general blurring and enlargement of the images ("haziness" or "fuzziness") without lateral movement.
3. Vibration and blurring combined.

Each of these effects is due to a characteristic change in the nature of the atmospheric disturbance. The first is produced by a regular and progressive change in density from one side to the other of the column of air through which the light has to pass to reach the telescope objective. This produces a general turning of the whole beam; the same effect as would be produced by a thin *prism* of glass placed in the path of the beam. If the equivalent "air prism" is perfectly homogeneous, we would have a displacement of the image simply, without *any* loss of definition (save the very slight effect due to the chromatic aberration of the "air prism"); this, however, as might be expected, very rarely happens, particularly if the objective is of any considerable size, since the "air prism" is being continually broken up and reformed with the refracting edge in a new direction. In many cases the "air prism" is formed over only a part of the the lens, in which case we have a "ray" shot out from the image in a direction at right angles to the refracting edge of the "prism."

Effects of the second kind may in the same way be considered to be due to the momentary interposition in front of the objective of air clots of more or less globular form, varying in density from the center outward.

The effect of these is to momentarily change the focus of the telescope, just as a thin *lens* of glass would do. Here again the effect is rarely a pure one, the "air lens" being irregular in

structure. If it were regular in structure and persistent in position and form, its effect could be almost completely compensated by a change in focus. And it is interesting to note that when the atmospheric disturbances are mainly of this nature the most frequent intervals of "good seeing" may be obtained when the eyepiece is set either a little inside or a little outside the position which would be best if the air were undisturbed, *i. e.*, inside, where the "air lenses" most frequently formed are thickest (most dense) at the center; outside, where the reverse is the case.

It is easy to see that the third effect is produced either by momentary combination of "air lenses" and "air prisms," or by "air lenses" alone, when the latter are not concentric with the axis of the telescope.

In considering the effect of these various disturbances on the intensity of the focal plane image, we can see in a general way that the relation between the magnitude of the effect and the dimensions of the object glass will be decidedly different in cases 1 and 2. In 1 there will be no loss in the intensity in case of visual observations, but in prolonged and continuous photographic exposures the loss will be (for small sources) directly proportioned to the square of the focal length and independent of aperture. In case 2 there will be a loss in both visual and photographic work (except at the instants of "best seeing"), which is proportional to the square (and in some cases to a higher power) of the aperture, but which decreases in proportion as the focal length increases. In 3 again we have a combination of both effects. In visual observations 2 is the most important; in prolonged photographic exposures 1 is the one to be chiefly considered.

YERKES OBSERVATORY.
Dec. 24, 1897.

(*To be continued.*)

Minor Contributions and Notes.

A NOTE ON THE DISCOVERY OF AN ERROR IN THE PAPERS OF STRUVE AND LORD RAYLEIGH, DEALING WITH THE APPLICATION OF THE PRINCIPLES OF THE WAVE THEORY TO THE DETERMINATION OF THE INTENSITY OF THE IMAGES OF FINE LINES AND EXTENDED AREAS AT THE FOCUS OF A TELESCOPE.[1]

THE general expressions for the intensity at various points in the images of point, line, and surface sources, as formed by a telescope of symmetrical aperture (rectangular, triangular, elliptical, or circular) have been developed on the principles of physical optics by Airy Stokes, Lommel, André, Struve, and most fully and comprehensively of all by Lord Rayleigh, whose memoir on the general principles of the wave theory[2] is a standard of reference for all workers in the subject. These results rendered possible for the first time the development of a satisfactory theory of the resolving power of optical instruments, which is the only logical basis upon which to discuss their comparative merits and efficiency with reference, either to ACCURACY (metrological or measuring power), or RESOLUTION (separating power); the two properties which chiefly concern us, in what may, for want of a better

[1] This error was discovered about three weeks ago when I resumed the investigation (temporarily laid aside by reason of the pressure of work attending dedication determination of color curve, and computing of photographic correcting lens for 40^{in} telescope. etc.), of the "Conditions of Maximum Efficiency in Astrophotographic Work," of which Part I, "General Theory of Telescopic Images," was published in the August number of this JOURNAL; and of which Part II, "General Effect of Atmospheric Aberration on the Intensity of Telescopic Images," appears elsewhere in the present number. The present note was not completed in time for insertion in full in the December number, and a short note simply announcing the discovery of the error was inserted in its place. Since then two articles on the same subject have appeared, one by Professor Schaeberle in the *Astronomical Journal* (No. 421), and the other by Mr. Newall in the *Monthly Notices* (November 1897). Both of these articles, however contain themselves a number of errors which have been pointed out in two communications addressed to the respective journals in which they appeared. I have therefore, allowed the present note to stand as it was originally written without reference to either Professor Schaeberle's or Mr. Newall's articles.—F. L. O. W., January 3.

[2] "Wave Theory," *Enc. Brit.*, 9th ed. **24**.

term, be called *quantitative* observations. More recently these same results, together with others deduced by similar methods, have been used in developing the theory of "contrast" and "delineating power," which, in conjunction with the older theory of resolving power, has been made the basis for comparing the efficiency of optical instruments with reference to DEFINITION (defining power): the property with which we are chiefly concerned in what we may call (in contradistinction to the term already used) *qualitative* observations.

The full discussion of "the conditions of maximum efficiency" in both classes of observations, as determined by considerations of resolving and delineating power, is of the greatest value and importance in indicating the methods of work and the forms of instruments calculated to secure the best results in any given line of research.

My attention was first directed to the investigation of the conditions of maximum efficiency in the use of the spectrobolometer, by my desire to obtain the best possible results in the investigation of the infra-red spectrum at the Smithsonian Astrophysical Observatory, and by my having been unable to find at the time this work was put into my hands any complete and satisfactory discussion of many of the various questions involved.[1] Subsequently my field of investigation was greatly extended by my having been placed in charge of the design and construction of the instruments and apparatus for the various lines of spectroscopic, spectrographic, and astrophotographic research at the Yerkes Observatory. A number of papers embodying the results

[1]At that time the only papers that had been published (as far as the writer is aware) in which both the general theory and practical design of optical instruments had been taken up and discussed satisfactorily by the methods of physical optics were those of Lord Rayleigh, whose results were derived with special reference to *laboratory* spectroscopes (see particularly "On the Manufacture and Theory of Diffraction Gratings, *Phil. Mag.*, **47**, 5, 1874; and "Investigations in Optics with Special Reference to the Spectroscope," *ibid.*, **8**, **9**, 1879–80); and of Professor Michelson, who had investigated the theory and pointed out a number of applications of the *interferometer* to astronomical measurements ("Interference Phenomena in a New Form of Refractometer," *Phil. Mag.*, **46**, 395, 1882; "Measurement by Light Waves," *Amer. Jour. Sci.*, **39**, 115, 1890; "Application of Interference Methods to Astronomical Measurements," *Phil. Mag.*, **30**, 1, 1890; "Application of Interference Methods to Spectroscopic Measurements," *ibid.*, **31**, 338, 1891, and **34**, 280, 1892, etc.). Since that time the theory of the microscope has been fully treated by Lord Rayleigh ("On the Theory of Optical Images with Special Reference to the Microscope," *Phil. Mag.*, August 1896), and the theory of the spectroheliograph by Michelson (*Ap. J.*, **1**, 1, January 1895). Several papers on the theory of the telescope have also been recently published by Strehl in the *Zeit. für Instrumentenkunde.*

of these investigations as they have progressed have been published during the last three years in this and other journals, and the conclusions that have been reached have been in general, I believe, accepted by astronomers and astrophysicists.[1]

But there is one result which has recently been used by the writer in developing the theory of contrast and "delineating power" that has been (as it now appears rightly) questioned. This is the result for the illumination of the focal plane of a telescope due to an infinitely extended luminous area, which was first announced (I believe) several years ago by Lord Rayleigh in his great memoir on the wave theory. After reviewing the work of Stokes and Struve, both of whom had investigated closely related problems (exactly the *same* problem from a mathematical point of view), he concludes as follows (§ 12):

"If we integrate (30)" . . . *i. e.*,

$$d\xi \int_{-\infty}^{+\infty} I_1^2 \, d\eta \qquad (30)$$

"with respect to ξ between the limits $+\infty$ and $-\infty$, we obtain πR^2 as has already been remarked. This represents the whole illumination

[1] For convenience in future reference the following partial list of these papers is here given: "Electric Control and Governors for Astronomical Instruments," *A. and A.*, **13**, 265, 1894; "An Improved Form of Littrow Spectroscope," *Phil. Mag.*, **37**, 137, 1894; "Fixed Arm Spectroscopes," *ibid.* **38**, 337, 1894; "General Considerations Respecting the Design of Astronomical Spectroscopes," *Ap. J.*, **1**, 52, 1895; "Design of Electric Motors for Constant Speed" (for astronomical instruments), *ibid.* **1**, 1895; "New Designs of Combined Grating and Prismatic Spectroscopes of the Fixed Arm Type and a New Form of Objective Prism," *ibid.* **1**, 232, 1895; "A Multiple Transmission Prism of Great Resolving Power," *ibid.* **2**, 264, 1895; "Fixed Arm Concave Grating Spectroscopes," *ibid.* **2**, 370, 1895; "The Use of a Concave Grating as an Analyzing Spectroscope," *ibid.* **3**, 47, 1896; "Further Notes on Astronomical Spectroscopes," *ibid.* **3**. 176, 1896; "The Conditions of Maximum Efficiency in the Use of the Spectrograph," *ibid.* **3**, 321, 1896; "The Objective Spectroscope," *ibid.* **4**, 54, 1896; "A New Form of Fluid Prism and Its Use in an Objective Spectroscope," *ibid.* **4**, 274, 1896; "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Mem. Spectr. Ital.*, **26**, 2, 1897, *Phil. Mag.*, May, *Wied. Ann.*, June, and *Jour. d. Phys.*, August 1897; "On the Conditions which Determine the Ultimate Optical Efficiency of Methods of Observing Small Rotations," *Phil. Mag.*, **44**, 83, 1897; "The Effect of the General Illumination of the Sky on the Brightness of the Field at the Focus of a Telescope," *M. N.*, June 1897; "On the Conditions which Determine the Limiting Time of Exposure of Photographic Plates in Astronomical Photography," *A. N.*, No. 3439, *Knowl.*, August and September 1897; "On the Conditions of Maximum Efficiency in Astrophotographic Work. Part I. General Theory of Telescopic Images,"

over the focal plane, *or reciprocally the illumination at O*[1] *(the same as at any other point), due to an infinitely extended luminous area.*" Earlier in the same memoir (§ 11) Rayleigh makes a similar statement with reference to the intensity of illumination in the images of *linear* sources. On this point he says (in the case of a telescope with rectangular aperture), "If the image of a line be at $\xi = 0$" (the center line in the field), "the intensity at any point ξ, η, in the diffraction pattern may be represented by

$$\int_{-\infty}^{+\infty} I_1^2\, d\eta \qquad (8)$$

and again (for circular aperture), "If we integrate (8) for I^2, with respect to η, we shall obtain a result applicable to a linear luminous source" . . . .

Before using these results in the development of the theory of contrast I had obtained a seeming verification of them along a slightly different line of analysis. I first obtained three general expressions for the intensity at any point in the image of the most general form of radiating source (one of any form or extent, and having any distribution in intensity), of which the elements vibrate independently. These expressions [(13), (16), and (17), of my paper in the ASTROPHYSICAL JOURNAL, August 1897][2] were first applied to the special cases of a short line and a small circular area of uniform intensity, and correct expressions (24) (28) obtained for the intensity at the centers of the images of such sources. They were then applied to the cases of lines of infinite length and areas of infinite extent and uniform intensity (which are also evidently special cases of the general one already stated), and two other expressions (19) (31) were obtained which appeared, and were, at first, assumed to be identical with the corresponding integrals (8) and (30) given by Rayleigh. Then, having independently carried through the analytical work of integration by the methods of both Stokes, Struve, and Rayleigh (as

Ap. J., **6**, 119, 1897; "A Comparison of the Photographic and of the Hand and Eye Methods of Delineating the Form and Surface Markings of Celestial Objects," *Pop. Astron.*, **5**, 200, 1897; "On the Photography of Planetary Surfaces," *Obs'y* **20**, 303, 365, 404, 1897; "On the Effect of the Size of an Objective on the Visibility of Linear Markings on the Surface of a Planet," *Ap. J.*, No. 413, etc. When the investigations now on hand are completed the writer hopes to publish these papers in a collected form.

[1] *O* is the center of the field; the origin of coördinates for ξ η.

[2] See paper, "General Theory of Telescopic Images," pp. 128–9.

given in Rayleigh's memoir), and not finding any error in the mathematical part of the work, I felt confirmed in the belief that the results reached were correct. And, indeed, although they seem at first sight sufficiently startling and contradictory to our established ideas on the matter (particularly the one for an extended uniform area), further consideration seemed to show that they were not altogether unreasonable. So far as the laws of geometrical optics were concerned they were at least no more in contradiction to them than the results obtained for the intensity in the physical images of point sources, or sources of very small angular magnitude, for which the ordinary geometrical laws are so greatly in error that the actual intensity at the centers of the images of such sources is in some cases less than 2 per cent. of what it would be on the laws of geometrical optics.[1] Naturally less discrepancy between the results obtained on the geometrical and on the physical theories would be expected in the case of very large areas than in the case of very small ones, but considered at least approximately (and I must confess that in general I am inclined, perhaps too much so, to look upon the results of the geometrical theory as at best but approximations) the result reached for an infinitely extended area seemed explicable under both theories. Geometrically considered, the image of an infinite area is itself infinite in extent, and since the intensity of illumination at any point of the image is geometrically proportional to the area of the objective divided by the area of the image, and since this latter quantity is constant (there being no question of infinities of different orders) the intensity would likewise be constant and proportional simply to the area of the objective.[2]

On the ground of the physical theory the difficulty of explaining the result seems even less, because we know that the illumination at the center of the field (which is the same as at any other point) is due to the effect not only of that portion of the source which would correspond to the geometrical image, but to all the outlying portions as well; even (in some degree) to those that lie at an angular distance of 90° from the optical axis. The effect of each one of these outlying

[1] In the case of many stars (whose angular magnitude must be less than 0″.001) it must be considerably less than 0.1 per cent., perhaps not more than 0.01 per cent. See table in *Ap. J.*, August 1897, p. 132.)

[2] The fallacy in this argument lies in the fact that the focal surface is not a plane, but (approximately), a portion of a sphere of radius *f*.

elements (distant more than a few minutes of arc from the edge of the geometrical image) is, it is true, extremely (mathematically, differentially) small, but on the other hand the number of the elements, contributing each its effect, is infinitely large. *A priori* there is no reason for considering that the summation of an infinite number of infinitely small effects may not be itself finite; indeed, it generally is. And if the effect of these outlying elements was finite it would (since the integration extends from $-\infty$ to $+\infty$) be independent of the distance, from the diffracting aperture, of the point at which the effects are summed up, *i. e.*, of the focal length of the telescope. When such a result was reached, then, the writer was not prepared to dispute its correctness, particularly after it had been accepted and announced by such an authority as Lord Rayleigh.

In addition to this there was another strong apparent confirmation of this mathematical result, in the complete and satisfactory explanation which it seemingly offered of the remarkable experimental results obtained by Professor Barnard with lenses of small aperture and short focal length. The striking agreement between the observed times of exposure required to obtain a given result with different lenses, and the computed times as determined by the theory of contrast (in the development of which this result had been used), was considered to be too remarkable and exact to be accidental and fortuitous.

For all these reasons, but chiefly for the last, the result for the focal plane illumination, due to an infinitely extended area, was at first regarded as well established, although doubts of its correctness were freely expressed by a number of astronomers and astrophysicists (among whom I may mention Keeler, Hale, Runge, Deslandres, Schaeberle, and Lord). No one with whom I discussed the matter was, however, able to point out any flaw in the reasoning or in the mathematical analysis by which the result was obtained.

It was only recently[1] that I was able to take up this investigation again with special reference to the general effect of atmospheric aberration on the intensity of telescopic images. When I found that the general effect of this, in prolonged photographic exposures was to diminish the *effective photographic intensity* of point and small surface sources in the ratio $\frac{1}{f^2}$ and thus, if the first result for the focal plane illumination, due to the sky, had been correct, make the *effective photographic*

[1] See preceding footnote.

contrast between field and such sources vary as $\frac{1}{f^4}$,[1] instead of $\frac{1}{f^2}$, as found in practice (and as found at first in theory); I was led to suspect an error somewhere in Rayleigh's work, and to discover, on a reinvestigation of the whole problem, where it lay.

In Rayleigh's case the error was made in assuming (without proof) that the general principle of "reciprocality" or "reversibility" of images was applicable to the problem under consideration. The integral

$$\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} I_1^2 \, d\eta \tag{32}$$

in which the integration is extended *over the whole of the focal plane*, is, it is true, the same in form as the integral

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} I_1^2 \, dy \tag{31}$$

in which the integration is extended *over the whole of an infinitely extended uniformly luminous area;* but they are not identical; in other words, although the first correctly represents the total illumination over the whole of the infinitely extended focal plane, due to a point source, *it does not* "reciprocally represent the illumination at a point due to an infinitely extended luminous area." My own error was essentially the same, *i. e.*, the assumption of the identity of the two integrals (31), (32);[2] but in making this assumption I also committed an analytical blunder, which, under the circumstances, seems well-nigh inexcusable. The two integrals would be identical (the limits being infinity) if the variables x, y, were symmetrically involved with the variables ξ, η, in the expression for I^2. And although this may appear at first sight to be the case, an inspection of equation (14),[3] which expresses the value of r (the variable in I^2) in terms of x, y, ξ, η, shows that this is not so. The two variables x, y, are each multiplied by a factor of dissymmetry $\frac{f}{D}$. In order to obtain an expression, of which the integral part is identical with the integral (32), we must introduce in (31) two new variables

[1] See paper: "Conditions of Maximum Efficiency in Astrophotographic Work, Part II. General Effect of Atmospheric Aberration on the Intensity of Telescopic Images," p. 70, of the present number of this JOURNAL.

[2] See equation (1), *Monthly Notices*, **57**, 587, June 1897.

[3] *Ap. J.*, **6**, 132, August 1897.

$$\xi_1 = \frac{f}{D} x. \quad \eta_2 = \frac{f}{D} y.$$

Substituting in (31) we obtain

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} I_1^2 dy = \frac{D^2}{f^2} \int_{-\infty}^{+\infty} d\xi_1 \int_{-\infty}^{+\infty} I_1^2 d\eta_1 = \frac{D^2}{f^2} (Z_{III}) \tag{31a}$$

and similarly in (19), (the expression for the intensity in the images of long lines),

$$\int_{-\infty}^{+\infty} I_1^2 dy = \frac{D}{f} \int_{-\infty}^{+\infty} I_1^2 d\eta_1 . \quad = \frac{D}{f} (Z_{II}). \tag{19a}$$

The integrals (Z_{III}) and (Z_{II}) are now respectively identical with (32) and (20) of my paper, which again are the same as the ones (30_R) and (8_R) used by Lord Rayleigh.[1]

These integrals were correctly evaluated in the preceding papers. The correct expressions for the intensity in the images of an infinitely extended uniformly luminous source, and of a long uniformly luminous line (at the center of the diffraction pattern) are therefore respectively

$$i_{III} = \text{const.} \frac{b^2}{f^2} = \text{const.}\ \beta^2 \tag{33a}$$

$$i_{II} = \text{const.}\ b\,\beta^2 \tag{23a}$$

or the same as the intensity at the centers of the images of areas and lines of finite dimensions (28) and (24).

And the expressions for the theoretical contrast (contrast under perfect atmospheric and instrumental conditions) as given in the paper, "General Theory of Telescopic Images," Part I, are modified by the introduction of a factor (f^2) in the numerator in cases (A), (B_2), and (C), and a factor f in (B_1), as elsewhere indicated. At the same time the expressions for the *practical photographic contrast* (during prolonged exposure) remain (owing to the introduction of the factors $\frac{1}{f}$ and $\frac{1}{f^2}$, expressing the effect of atmospheric aberration during such exposures) practically identical with the expressions (36), (39), which have been used in the detailed consideration of cases (A) and (C) from the astrophotographic side. ("On the Conditions which Determine the Limiting Time of Exposure of Photographic Plates in Astronomical Photography," *A. N.*, No. 3439, and *Knowledge*, August and Septem-

[1] "Wave Theory," *Enc. Brit.*, **24**, §§ 11 and 12.

ber 1897). The main conclusions reached in the preceding paper, dealing with these particular cases are, therefore, correct (if the general theory of contrast upon which these conclusions are based is correct, and this has not, I believe, been questioned), as well as those in my preliminary note to the R. A. S. ("The Effect of the General Illumination of the Sky on the Brightness of Field at the Focus of a Telescope," *M. N.*, June 1897). One of the minor conclusions of the first paper (*A. N.*, 3439) was based directly on the erroneous result for I^2_{111} and is consequently wrong. This is the conclusion with respect to the "fogging" of photographic plates by the light from the sky. But this result was not considered at the time it was obtained as of particular importance [as may be seen from the following quotation, § 100: "The mere ability to lengthen the time of exposure (at least beyond twenty-four hours) by decreasing the size of the photographic objective would not, in itself, be of great importance because there would be too much risk and difficulty in accurately following an object for a much greater length of time than is done at present"], and it was not, therefore, included in the summary of conclusions at the end of the paper.

The only other case yet considered in detail is case *D*, for which the expression for the theoretical contrast (40) was correct, as originally derived, and in the consideration of which the effect of atmospheric aberration was fully taken into account.[1] It was very unfortunate that this effect was not also considered in detail in cases (*A*) and (*C*) at the very first, as it was such consideration that led later to the discovery of the error in Rayleigh's result. But although the effect had not by any means been lost sight of, (having already been considered in the theory of the spectroscope and spectrograph,[2] as well as in case (*D*) already referred to), its importance was at first underestimated, and its consideration therefore deferred to the second part of my general paper, "Conditions of Maximum Efficiency in Astrophotographic Work," which, as already indicated, appears elsewhere in the present number.

F. L. O. Wadsworth.

Yerkes Observatory,
December 16, 1897.

[1] See papers: "On the Photography of Planetary Surfaces," *Obs'y* **20**, 333, 365, 404; and "The Effect of the Size of the Objective on the Visibility of Linear Markings on the Planets," *Ap. J.*, **413**.

[2] See particularly *Ap. J.*, **4**, 59, 60, June 1896.

THE PHOTOGRAPHIC NORMAL SOLAR SPECTRUM.

CONSECUTIVE WAVE-LENGTH EDITION.

SHORTLY after the issue of my "Photographic Studies of the Solar Spectrum," several leading men of science on both sides of the Atlantic expressed opinions in favor of a map on the same lines in regard to the various aspects under different conditions of the atmosphere and of solar altitude, but in consecutive wave-length parts for convenience of general reference.

Selections were submitted for consideration or approval, but owing to the fact that a number contained two orders for the purpose of showing the relative wave-lengths, it was felt that the resulting want of uniformity would still be an objection.

For this and other reasons the work has been entirely remodeled, and although the production of a uniform series was the primary object, no pains have been spared to make such improvements as seemed to be necessary, not only in respect to the general appearance and correctness of delineation, but in the introduction of some new features, which it is hoped will prove of service.

The work consists of thirty-eight sections, eight of which may be regarded as supplementary. Each section contains two or more subjects, and measures $8\frac{1}{2} \times 3\frac{1}{4}$ inches. The scale and other numbers have been written with a fine point and are not obtrusive enough to mar the subject.

The degree of enlargement requiring mm. units for the wave-length scale has been found sufficient for the visible and infra-red portions. In adopting $1\frac{1}{3}^{mm}$ as units for the violet end, the ratio of the two units is the same as that existing between the 3d and 4th orders.

No feature discernible in the original negatives under microscopical examination is wanting in the finished prints. A hand magnifier on the supplementary parts may, however, be required to reveal the structure of dense groups or close doubles. With the same object in view, short exposures have been given on the margins of plates for the region between wave-lengths 3200 and 5000.

With the exception of two or three supplementary parts, the ordinary method of enlarging has been employed, and is referred to in the descriptive supplement.

INDEX.

10 SCALE DIVISIONS = $13\frac{1}{2}^{mm}$.

Where the same remarks are applicable to both editions the numbers of the first edition are referred to in the third column and on the prints themselves.

Section	No.	Reference	Solar altitude	Wave-lengths	Including
A	1	21	55	2988:3149	*t*, T, *s*, S, *r*
	2	22	53	3124:3285	*r*, R
B	3	37	51	3196:3357	Q
	4	38	51	3286:3446	P, O
C	5	46	49	3392:3552	O
	6	49	41	3468:3628	N
D	7	50	45	3560:3720	N
	8	63	31	3643:3803	M
E	9	73	..	3740:3900	L
	10	74	..	3815:3975	K, H
F	11	77	..	3887:4047	K, H
	12	78	..	3958:4118	H, *h*
G	13	83	.	4052:4212	*h*
	14	..	..	4187:4347	G, *g*

10 SCALE DIVISIONS = 10^{mm}.

Section	No.	Reference	Solar altitude	Wave-lengths	Including
H	15	39	..	4263:4478	G, *g*
	16	40	50	4381:4595	
I	17	47	49	4522:4736	
	18	48	49	4625:4839	
J	19	51	45	4747:4961	F
	20	52	31	4857:5071	F
K	21	66	47	5040:5255	*b*
	22	75	48	5085:5300	*b*, E
L	23	80	..	5222:5436	E
	24	82	32	5402:5616	
M	25	85	34	5513:5727	
	26	86	6	5597:5811 Low sun	
N	27	..	20	5672:5886 Moist	
	28	9	10	5682:5896 Low temperature	

INDEX—*continued.*

10 SCALE DIVISIONS = $13\frac{1}{2}^{mm}$.

Section	No.	Reference	Solar altitude	Wave-lengths		Including
n	29	7	28	5688:5902		D
	30	10	10	5706:5920	Doppler effects	D
O	31	15	13	5844:6058		D
	32	18	1	5844:6058	Rising sun	D
P	33	13	20	5777:5991	Moist	D
	34	25	40	5983:6197		
Q	35	27	43	6160:6374		α
	36	29	38	6310:6524		
R	37	28	7	6188:6402	Low sun	α
	38	..	7	6398:6612	Low sun	C
r	39	30	45	6390:6604		C
	40	31	12	6390:6604	Low sun	C
s	39A	..	9	6390:6604	Low sun	C
	41A	..	9	6498:6713	Low sun	C
S	41	..	45	6498:6712		C
	42	42	32	6686:6900		B
T	43	43	37	6840:7054		B
	44	44	7	6850:7064	Low sun	B
U	45	53	33	7015:7229		*a*
	46	55	50	7145:7359		*a*
V	47	57	11	7100:7314	Low sun	*a*
	48	57	11	7255:7469	Low sun	
W	49	56	44	7296:7510		
	50	67	44	7405:7619		A
X	51	69	60	7555:7762		A
	52	58	11	7520:7734	Low sun	A
Y	53	71	52	7573:7787		A
	54	69	60	7759:7973		
Z	55	70	52	7928:8142		
	56	70	52	8132:8346		Z

SUPPLEMENTARY SECTIONS.

Most of these are made up of four strips, and contain all the portions which abound with complex groups and close doubles. The first seven are under a wave-length scale of 2^{mm} units. The remainder consists of highly magnified isolated groups, together with subjects possessing points of special interest.

Section	No.	
a	1A 2A	3051 : 3158 Shows coincidences 3130 : 3237 with red spectrum of the first order
b	3A 4A	3196 : 3303 3296 : 3403
d	5, 6 7, 8	3396 : 3503 3496 : 3603 3596 : 3703 3696 : 3803
f	9, 11 12, 13	3793 : 3900 3898 : 4005 4003 : 4110 4106 : 4213
h	14, 15 16, 17	4211 : 4318 4316 : 4423 4421 : 4528 4526 : 4633
j	18 19, 20	4629 : 4736 4732 : 4839 4838 : 4945 4944 : 5051
o	31, 32 31, 32	5845 : 5952 } Comparison high and low sun (cylindrical lens) 5926 : 6033 }
p	. . . . .	Groups 3880, 5007, 5205, 5270, 5276, 5603, 5884, 6103, and 6164. Magnified 12 : 18 times. (Cylindrical lens)
q	1 2 3 4	5914 : 6870 8228 Groups 7130 ; 7740 Low sun ; low temperature ; dry 7130 : 7740 Low sun ; high temperature ; moist 7592 : 7699 A line, 2^{mm} scale. (Cylindrical lens)

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γ*	5, 6*	5520 : 6590 5880 : 6950
δ	7, 8	6680 : 7750 7280 : 8350

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GEORGE HIGGS.

TUEBROOK, LIVERPOOL,
December, 1896.

NOTE ON SUN DRAWINGS.

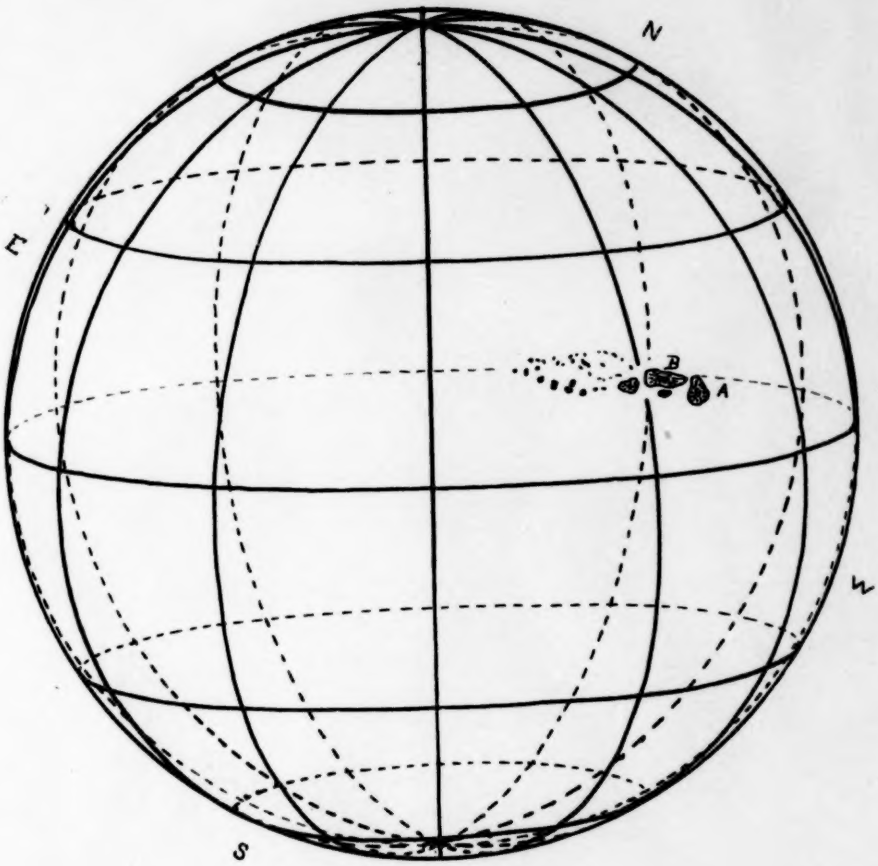
THE accompanying results were obtained by the well-known method of projection-drawing. The drawing was made by attaching a board in front of the eyepiece of a twelve-inch equatorial in such a manner that an image of the Sun of some convenient size could be formed upon it. In our case, an image of six inches diameter was chosen. A piece of drawing paper, upon which was drawn a circle of six inches diameter, was tacked to the board so that the center of the circle was in the line of collimation of the telescope. The focus and the distance of the board were then so adjusted that the image of the Sun should exactly fit the circle. An east and west line was determined by allowing the image to pass across the paper and tracing the path of some small spot. Then, keeping the image fixed by means of the driving clock, the outlines of the spots were traced. From the drawing thus made are taken the distance from the spot to the center of the circle, and its position-angle from the north and south line. By means of these data, the heliographic latitude and longitude of the spot may be calculated by Carrington's method.

The accompanying observations, of course, do not include all the spots which appeared between the given dates.

Plate I represents the Sun, as seen in the direct view September 18.83, 1896, Greenwich Mean Time (civil reckoning), showing a large group of spots, the position of the poles and the parallels and meridians for every thirty degrees. The parallels and meridians were drawn according to the method given by R. A. Proctor.

Plate II, Fig. 1, represents the Sun on January 7.80, 1897, showing conspicuously the large spot P. Fig. 2, February 10.85, 1897, shows the same spot P again, after it has completed more than a revolution. Figs. 3, 4, 5 and 6 represent, respectively, the solar appearance on November 7.74, 9.83, 13.74, and 14.55, 1896. These show nicely the motion of the large spot G and the changes in the smaller spots.

PLATE I.



SUN SPOT GROUP SEPT. 18.83, 1896.

PLATE II.

N

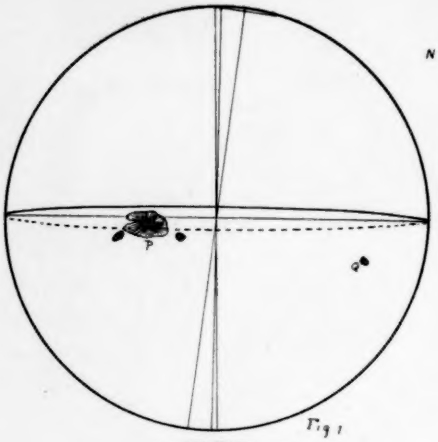


Fig 1

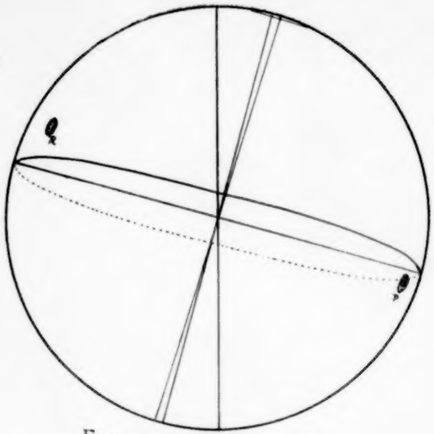


Fig 2

E

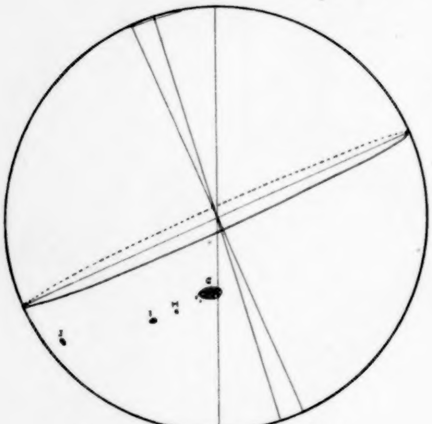


Fig 3

W

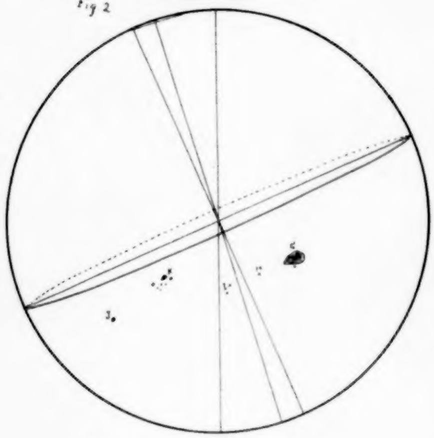
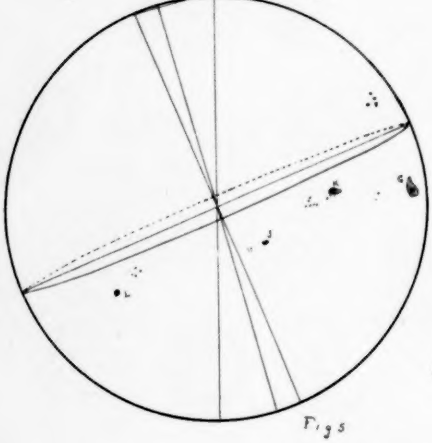


Fig 4



S

Fig 5

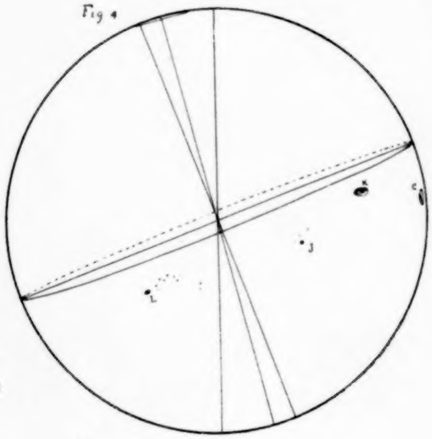


Fig 6

Gr. Mean Time (civil)	Group	Spot	Dist. from cente in inches	$\frac{r}{R}$ corrected for distortion	Position angle	Sun's radius	Sun's longitude	Heliographic latitude of spot	Longitude from node	Heliographic longitude
1896										
Sept. 18.83	1	A	1.85	.633	32° 305′	15.′ 97	176° 23	10° 19′	321° 38′	59° 5′
	1	B	1.60	.549	307 0			12 31	315 23	52 50
Sept. 19.80	1	A	2.37	.802	303 30	15. 97	177 20	10 58	336 49	60 30
	1	B	2.13	.724	307 0			13 30	329 43	53 24
	2	C_1	2.00	.682	101 45			14 13	240 5	323 46
	3	D	2.84	.952	128 0			−9 59	212 53	296 34
	3	E	2.89	.967	129 55			−12 22	210 8	293 49
Sept. 21.58	1	A	2.93	.980	303 30	15. 98	179 4	9 24	4 17	62 43
	1	B	2.80	.939	306 0			12 25	355 35	54 1
	2	C_1	.99	.342	92 5			14 22	265 59	324 25
	3	D	2.33	.789	134 10			−10 13	235 40	294 6
	3	F	2.61	.879	133 15			−12 6	226 13	284 39
Sept. 23.74	2	C_2	.59	.205	327 0	15. 99	181 11	12 58	276 40	304 28
	3	D	1.27	.438	157 30			−10 25	267 43	295 31
	3	F	1.68	.576	148 30			−12 11	257 27	285 15
Nov. 7.74	4	G	1.06	.367	176 0	16. 19	225 52	−15 36	321 44	71 14
	4	H	1.42	.489	155 20			−15 53	309 40	59 10
	4	I	1.70	.583	146 10			15 29	301 19	50 49
	5	J	2.80	.939	127 50			−12 27	263 23	12 53
Nov. 9.83	4	G	1.24	.427	243 30	16. 20	227 58	−15 43	350 37	70 29
	4	H	.95	.329	218 50			−15 10	339 15	59 7
	4	I	.94	.325	190 50			−15 18	329 52	49 44
	5	J	2.05	.697	132 10			−11 1	291 56	11 48
	6	K	1.10	.380	134 30			−5 5	313 9	33 1
Nov. 13.74	4	G	2.81	.943	274 45	16. 21	231 55	−14 58	44 15	68 39
	6	K	1.68	.576	277 10			−6 0	9 20	33 44
	5	J	.88	.304	236 0			−11 48	345 24	9 48
	7	L	1.88	.643	129 0			−8 50	297 4	321 28
Nov. 14.55	4	G	2 91	.973	275 0	16. 22	232 44	−15 21	53 53	66 48
	6	K	2.08	.707	280 0			−6 20	22 36	35 31
	7	L	1.45	.499	133 0			−9 2	310 38	323 33
	5	J	1.24	.427	254 30			−12 19	358 57	11 52
Dec. 5.74	8	M	.83	.288	319 25	16. 28	254 10	9 25	13 45	86 5
	9	N	1.08	.373	32 30			20 41	352 27	64 47
	10	O	2.67	.898	123 45			−17 35	297 31	9 51
Dec. 7.79	8	M	1.93	.658	297 30	16. 28	256 16	8 19	42 10	85 26
	9	N	1.30	.448	335 0			19 57	19 31	62 47
1897	10	O	1.90	.649	128 30			−15 28	324 7	7 23
Jan. 7.80	11	P	1.00	.345	94 50	16. 30	287 51	−5 56	13 13	336 37
	12	Q	2.13	.724	254 25			−12 53	79 16	42 40
Feb. 10.85	13	R	2.67	.898	62 15	16. 24	322 27	7 12	7 38	208 3
	11	P	2.83	.948	251 0			−4 44	139 40	340 5
Feb. 13.71	13	R	1.31	.452	45 25	16. 23	325 20	5 40	47 9	207 1
Feb. 17.87	13	R	1.50	.516	275 10	16. 22	329 31	5 49	103 19	204 10
eb. 20.71	13	R	2.75	.924	259 25	16. 21	332 23	5 26	144 18	204 52

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LADD OBSERVATORY, Providence, R. I.

November 1897.

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